Chapter 6

Statically Determinate Plane Trusses

This chapter starts with the definition of a truss and briefly explains various types of plane truss. The determinacy and stability of a truss also will be discussed. The procedures of analyzing statically determinate plane trusses will be developed by using the method of joints and the method of sections.

After successfully completing this chapter you should be able to:

- Determine the type of trusses and its application in construction
- Analyse the trusses members using related methods.

6.1 Introduction

- A truss is defined as an assemblage of straight members connected at their end by flexible connections to form a rigid configuration.
- If all the members of a truss and the applied loads lie in a single plane, the truss is called a plane truss.
- Because of their light weight and high strength, plane trusses are widely used, and their applications range from supporting decks of bridges and roofs of buildings.
- Modern trusses are constructed by connecting members, which usually consist of structural steel or aluminum shapes or wood struts, to gusset plates by bolted or welded connections.

6.2 Types of trusses and its application

- Typical framing systems for a roof supported by plane trusses are as shown in Figure 6.1.
- In this case, two or more trusses are connected at their joints by beams, termed as **purlins,** to form a three-dimensional framework.

- The roof is attached to the purlins, which transmit the roof load (weight of the roof plus any other load) as well as their own weight to the supporting trusses at the joints.
- Because of this applied loading acts on each truss in its own plane, the trusses can be treated as plane trusses.
- Some of the common configurations of roof and bridge trusses are shown in Fig. 6.2 and Fig. 6.3, respectively.

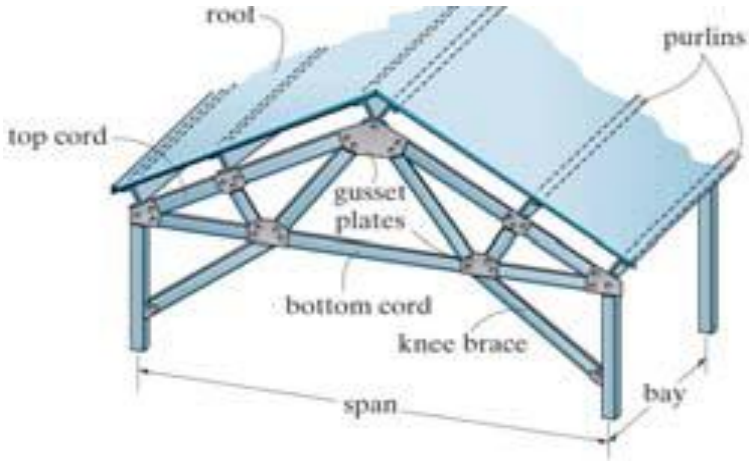


Fig. 6.1 Framing system of roof supported by plane truss

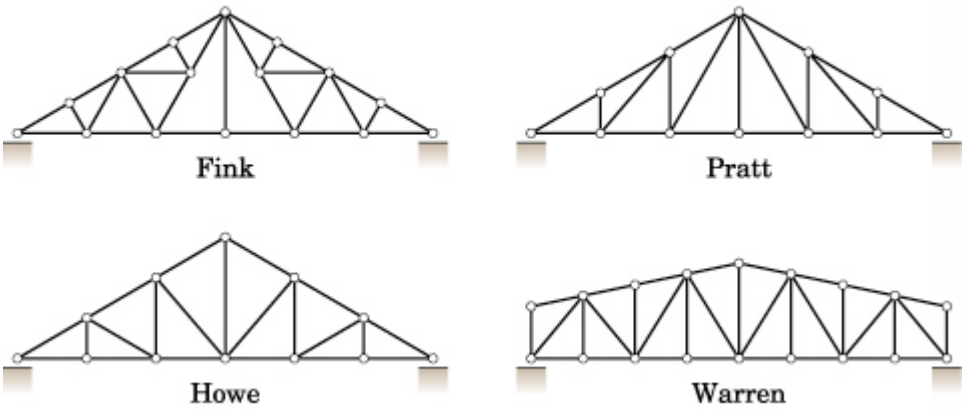


Fig. 6.2 Common types of roof truss

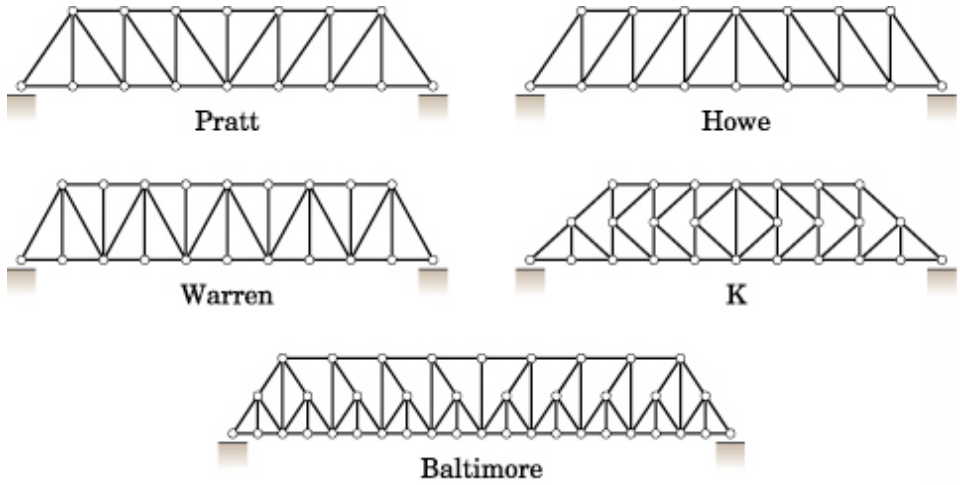


Fig. 6.3 Common types of bridge trusses

6.3 Assumptions for analysis of trusses

The analysis of trusses is usually based on following assumptions:

a. All members are connected only at their ends by frictionless hinges in plane trusses.

b. All loads and support reactions are applied only at the joints.

c. The centroidal axis of each member coincides with the line connecting the centers of the adjacent joints.

6.4 Simple Trusses

- The basic element of a plane truss is the triangle. Three bars joined by pins at their ends as shown in Fig. 6.4(a), constitute a rigid frame. The term *rigid* is used to mean non-collapsible and also to mean that deformation of the members due to induced internal strains is negligible.
- The basic truss element can be enlarged by attaching two new members BD and CD as in Fig. 6.4 (b). Four or more bars pin-jointed to form a polygon of as many sides constitute a non-rigid frame.
- The truss also can be extend by adding additional units of two end-connected bars, such as *DE* and *CE* or *AF* and *DF*, as in Fig.6.4 (c), which are pinned to two fixed joints. In this way the entire structure will remain rigid.

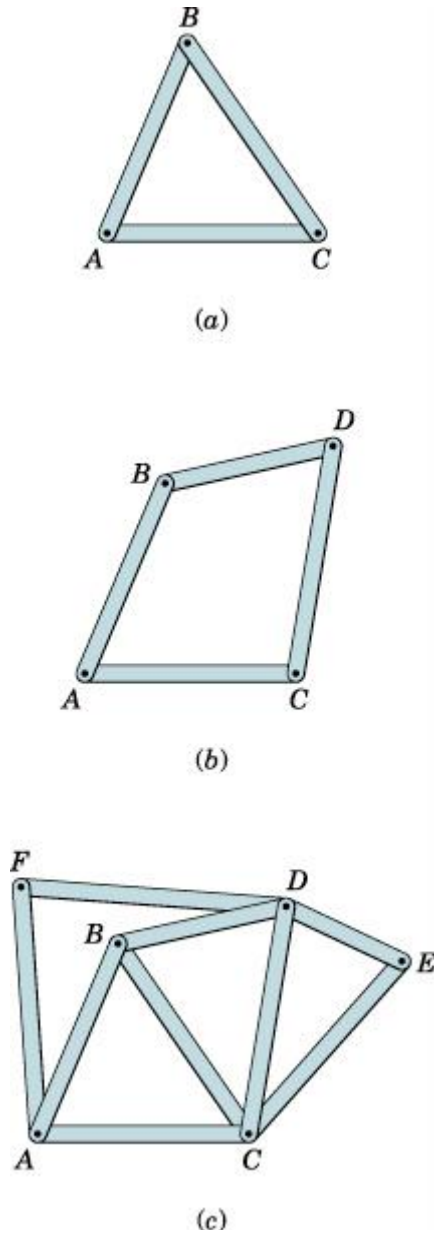


Fig. 6.4 Simple Truss

- Each member of a truss is normally a straight link joining the two points of application of force.
- The two forces are applied at the ends of the member and are necessarily equal, opposite, and *collinear* for equilibrium.
- The member may be in tension or compression, as shown in Fig. 6.5.

- The equilibrium of a portion of a two-force member, the tension *T* or compression *C* acting on the cut section is the same for all sections.

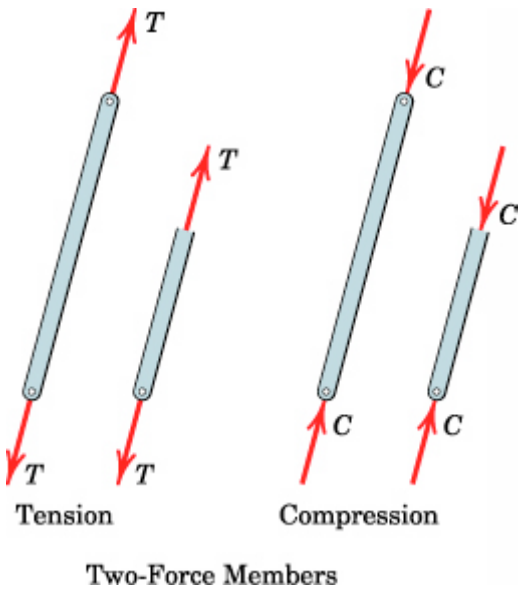


Fig. 6.5 The tensile and compressive force

6.5 Stability and Determinancy

- **Internal Stability** ≡ number and arrangement of members is such that the truss does not change its shape when detached from the supports.
- **External Instability** ≡ instability due to insufficient number or arrangement of external supports.

6.5.1 Internal Stability

- $m < 2j - 3$
  ⇒ truss is internally unstable

- $m \geq 2j - 3$
  ⇒ truss is internally stable provided it is geometrically stable

  where:

  m ≡ total number of members
  j ≡ total number of joints

- **Geometric stability** in the second condition requires that the members be properly arranged.
- **Statically Determinate Truss** - if all the forces in all its members as well as all the external reactions can be determined by using the equations of equilibrium.
- **Statically Indeterminate Truss** - if all the forces in all its members as well as all the external reactions cannot be determined by using the equations of equilibrium.
- **External Indeterminacy** - excess number of support reactions
- **Internal Indeterminacy ≡** excess number of members
- **Redundants ≡** excess members or reactions

Number of redundants defines the degree of static indeterminacy

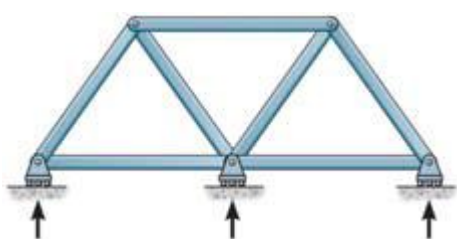
$m + r < 2j$
⇒ statically unstable truss

$m + r = 2j$
⇒ statically determinate truss

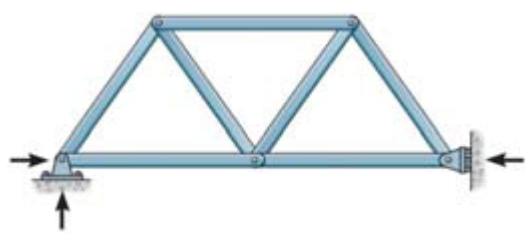
$m + r \geq 2j$
⇒ statically indeterminate truss

- A truss can be unstable if it is statically determinate or statically indeterminate.
- A truss is externally unstable if all of its reaction is concurrent or parallel.

<u>External Unstable</u>

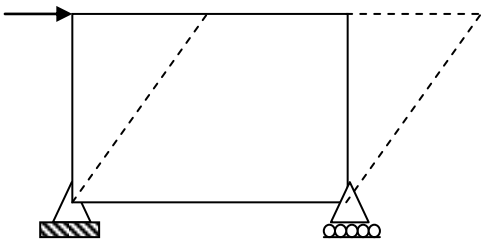


Unstable-parallel reactions

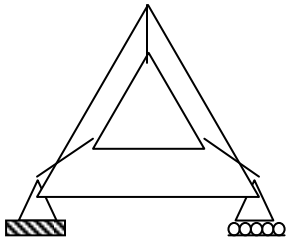


Unstable-concurrent reactions

Internal Unstable



4+3 < 2(4)

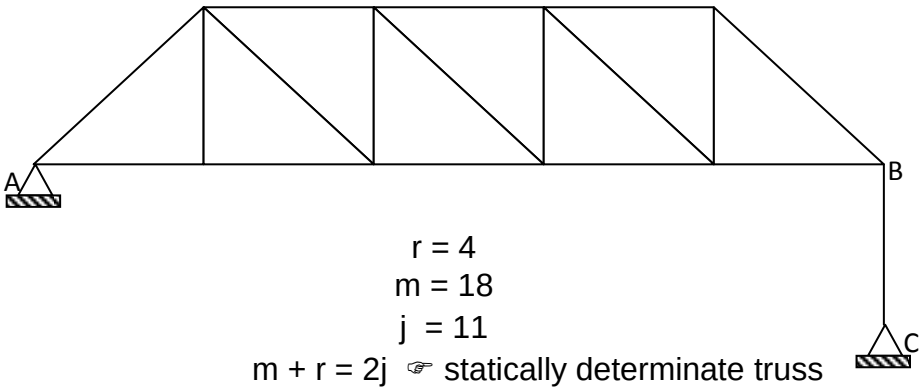


Concurrent at center point

Example 6.1

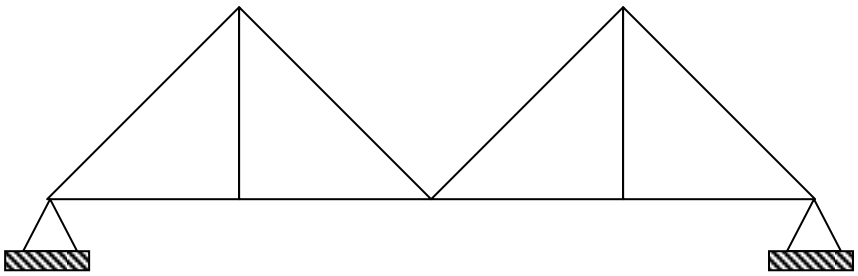
Determine the determinancy criteria for the truss as shown below:

(a)



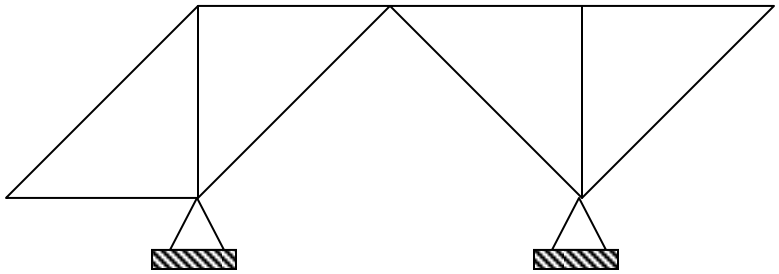
r = 4
m = 18
j = 11
m + r = 2j ☞ statically determinate truss

(b)



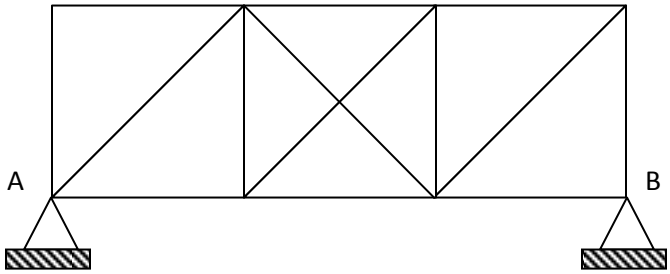
r = 4
m = 10
j = 7
m+r=2j ☞ statically determinate truss

(c)



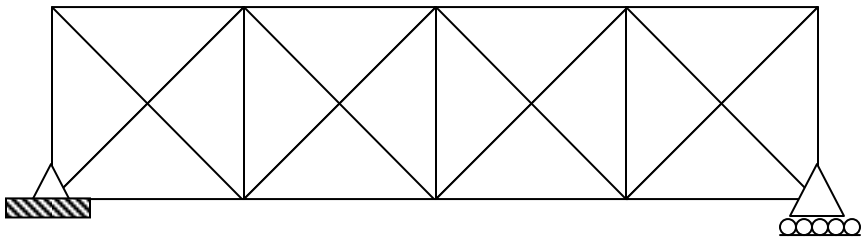
r = 4
j = 7
m=10
m+r=2j ☞ statically determinate truss

(d)



r = 4
m = 14
j = 8
m + r >2j ☞ statically indeterminate truss

(e)



r = 3
m = 21
j=10
m+r>2j ☞ statically indeterminate truss

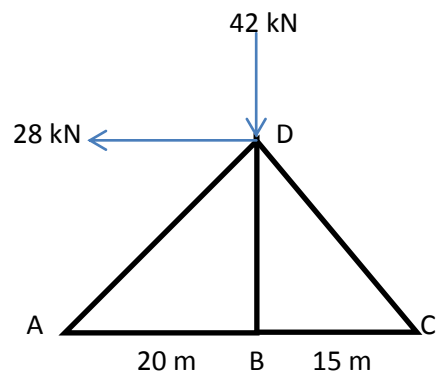
6.6 Method of Joints

- **Method of Joints**-the axial forces in the members of a statically determinate truss are determined by considering the equilibrium of its joints.
- **Tensile**(**T**) **axial member force**is indicated on the joint by an arrow pulling away from the joint.
- **Compressive**(**C**) **axial member force**is indicated by an arrow pushing toward the joint.

- When analyzing plane trusses by the method of joints, only two of the three equilibrium equations are needed due to the procedures involve concurrent forces at each joint.

Example 6.2

Calculate all member forces by using method of joints.



Solution

Determine the support reactions:

$\sum F_x = 0$

$H_A - 28 = 0$

$H_A = 28$ kN

$\sum M_c = 0$

$-R_A (35) + 28(20) + 42(15) = 0$

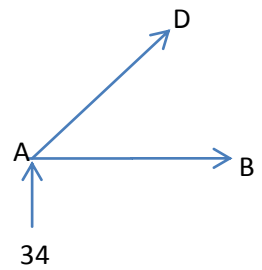
$R_A = 34$ kN

$\sum F_Y = 0$

$34 - 42 + R_C = 0$

$R_c = 8$ kN

<u>Joint A</u>



$\sum F_Y = 0$

$34 + \sqrt{2}\, F_{AD} = 0$

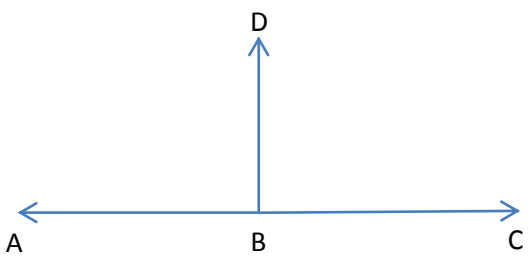
$F_{AD} = -48.08$ kN (C)

$\sum F_x = 0$

$28 - \sqrt{2}(48.08) + F_{AB} = 0$

$F_{AB} = 6$ kN (T)

<u>Joint B</u>



$\sum F_x = 0$

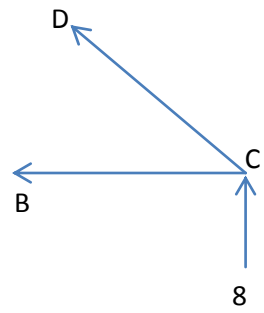
$-6 + F_{BC} = 0$

$F_{BC} = 6$ kN (T)

$\sum F_Y = 0$

$-F_{BD} = 0$

Joint C



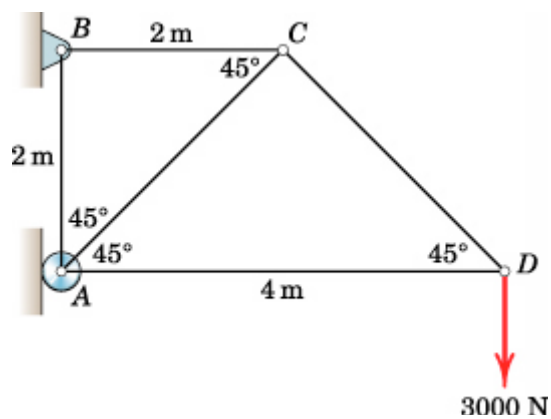
$\sum F_x = 0$

$-6 + \frac{3}{5} F_{CD} = 0$

$F_{CD} = 10$ kN (T)

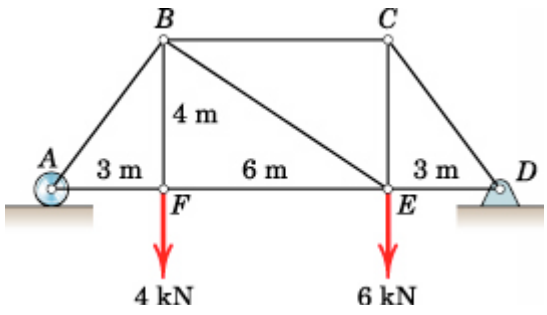
Exercise 6.1

1) Determine the force in each member of the loaded truss.



Answer:
AB = 3000 N T, AC = 4240 N C, CD = 4240 N T
AD = 3000 N C, BC = 6000 N T

2) Determine the force in each member of the loaded truss.



Answer:
AB = 5.63 kN C, AF = 3.38 kN T
BC = 4.13 kN C, BE = 0.901 kN T
BF = 4 kN T, CD = 6.88 kN C
CE = 5.50 kN T, DE = 4.13 kN T
EF = 3.38 kN T

6.6.1 Zero Force Members

(a) If only two noncollinear members are connected to a joint that has no external loads or reactions applied to it, then the force in both members is zero. (Fig. 6.6.1 (a))

(b) If three members, two of which are collinear, are connected to a joint that has no external loads or reactions applied to it, then the force in the member that is not collinear is zero.

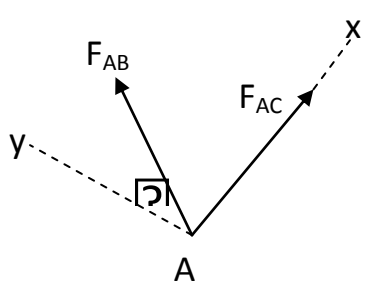


Fig. 6.6.1 (a)

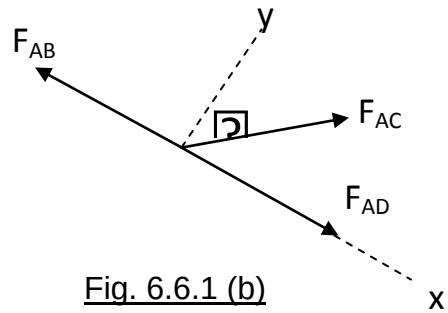


Fig. 6.6.1 (b)

6.6.1.1 Zero Force Member Calculations

Figure 6.6.1(a):

$\Sigma F_y = 0 = F_{AB} \cos$?
☞ $F_{AB} = 0$

$\Sigma F_x = 0 = F_{AC} + F_{AB} \sin$?
☞ $F_{AC} = 0$

Figure 6.6.1(b):

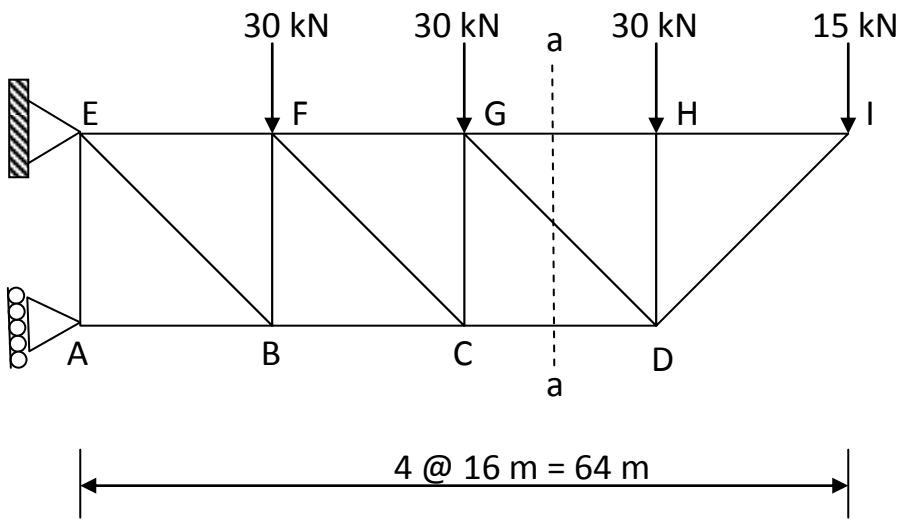
$\Sigma F_y = 0 = F_{AC} \cos$?
☞ $F_{AC} = 0$

6.7 Method of Sections

- This method involves cutting the truss into two portions (free body diagrams, FBD) by passing an imaginary section through the members whose forces are desired.
- Desired member forces are determined by considering equilibrium of one of the two FBD of the truss.
- This method can be used to determine three unknown member forces per FBD since all three equilibrium equations can be used.

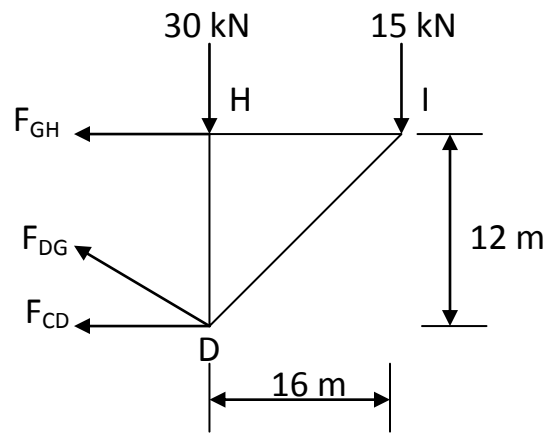
Example 6.3

Determine the forces of member GH, DG and CD by using method of section.



Solution:

- Calculate the support reaction if it is required.
- Cut the section of the truss through the members where forces are to be determined.
- Choose one section either left or right section.



$\sum M_D = 0$

$-15(16) + F_{GH}(12) = 0$
$F_{GH} = 20$ kN (T)

$\sum F_y = 0$

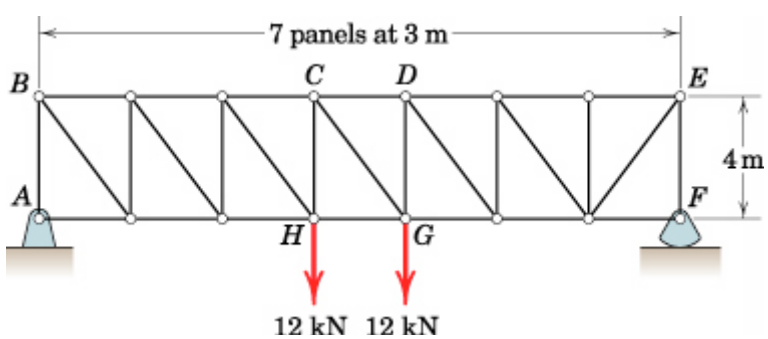
$-30-15+\frac{3}{5} F_{DG}=0$
$F_{DG} = 75$ kN (T)

$\sum F_x = 0$

$-20-\frac{4}{5}(75) - F_{CD} = 0$
$F_{CD} = -80$ kN

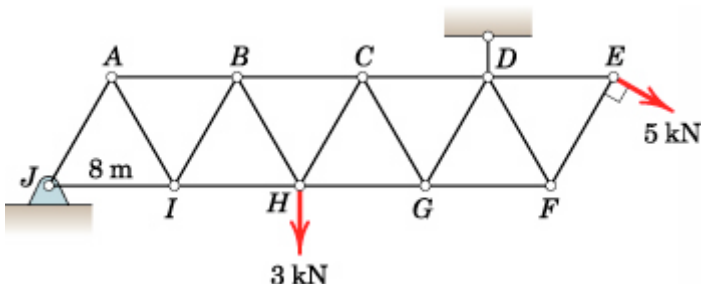
<u>Exercise 6.2</u>

1) Determine the forces in members *CG* and *GH*.



Answer:
CG = 0, GH = 27 kN T

2) Calculate the forces in members *BC*, *CD*, and *CG* of the loaded truss composed of equilateral triangles, each of side length 8 m.



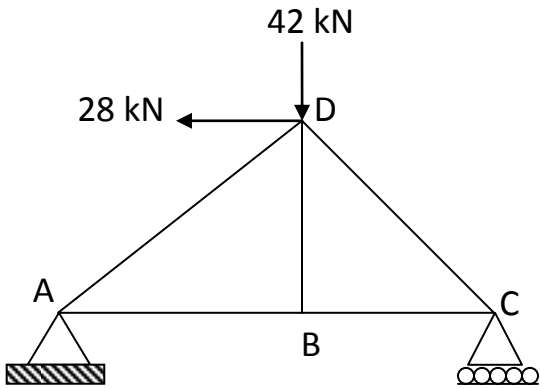
Answer:
BC = 1.155 kN T, CD = 5.20 kN T
CG = 4.04 kN C

6.8 Alternative Computation using Joint Equilibrium Method

- An alternative method can be applied to determine the member forces.
- The purpose is to reduce the time for calculation.

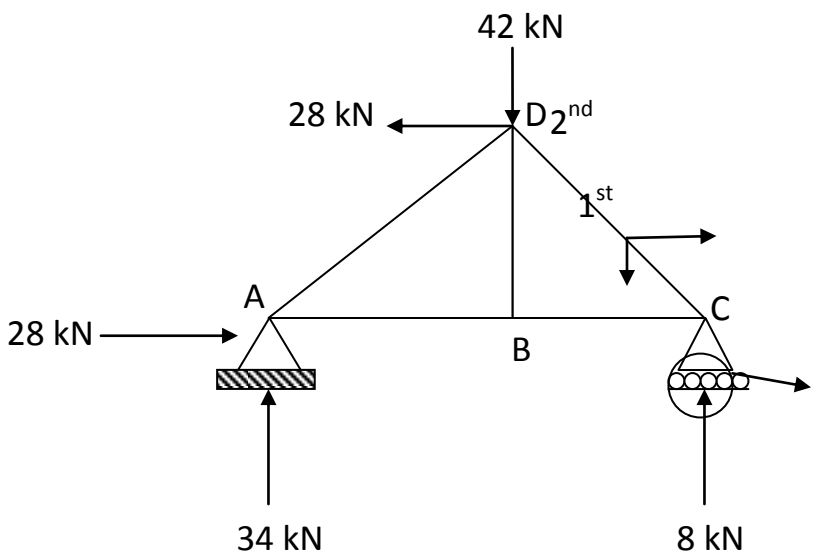
<u>Example 6.4</u>

Determine all the member forces.



Solution:

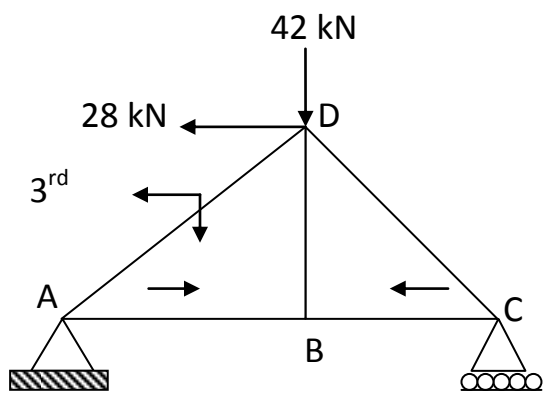
The calculation can be started at joint A or B.



1st

8/BD = x/EB
8/20 = x/15
x = 6

$F_{CD} = \sqrt{6^2 + 8^2}$
= - 10 kN



34/AB = x/BD
34/20 = x/20
x = 34

$F_{AD} = \sqrt{34^2 + 34^2}$
F_{AD} = - 48.08 kN