Chapter 6

PLASTIC ANALYSIS

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6.1 INTRODUCTION

- Generally, the plastic analysis is an alternative method to analysis the material of steel because of the ductile behaviour.

- The characteristic stress-strain diagram for a steel material is shown in ***Figure 6.1***. In structural analysis, the steel structure passes three distinct stages in the stress strain relationship as shown in ***Figure 6.1***. When the applied stress is proportional to the strain, a material is said to obey Hooke's Law. There is a linear relationship between the stress and strain; the steel is in the elastic behaviour. The material will return to its original undeformed shape if the load is removed. The slope is called the modulus of elasticity, *E* or Young's Modulus.

- In the second stage, as the stress increased further the plasticity spread inwards until an entire cross section of structure has yielded at point 1 as shown in ***Figure 6.1***. The yielded section creates a ***plastic hinge*** and lost all resistance to rotation. At this stage, the steel attain its maximum possible moment capacity called the ***plastic moment***, M_p. The development of the hinge caused a redistribution of the bending moments across the structure. The redistribution enables the structure to carry more loads after first hinge has formed.

- The second plastic hinge forms at the next most critical stage. On further increase in stress, the bending moments at the section of the two plastic hinges remain constant at their plastic moments and it keep increasing until the third plastic hinge forms. The process of the formation of successive plastic hinges continues until collapse of structure. The structure has now become a ***mechanism***.

- The purpose of plastic analysis is to determine the ***collapse load*** or ***ultimate load***. Plastic analysis considers the behaviour of structure in plastic limit before the structure collapse.

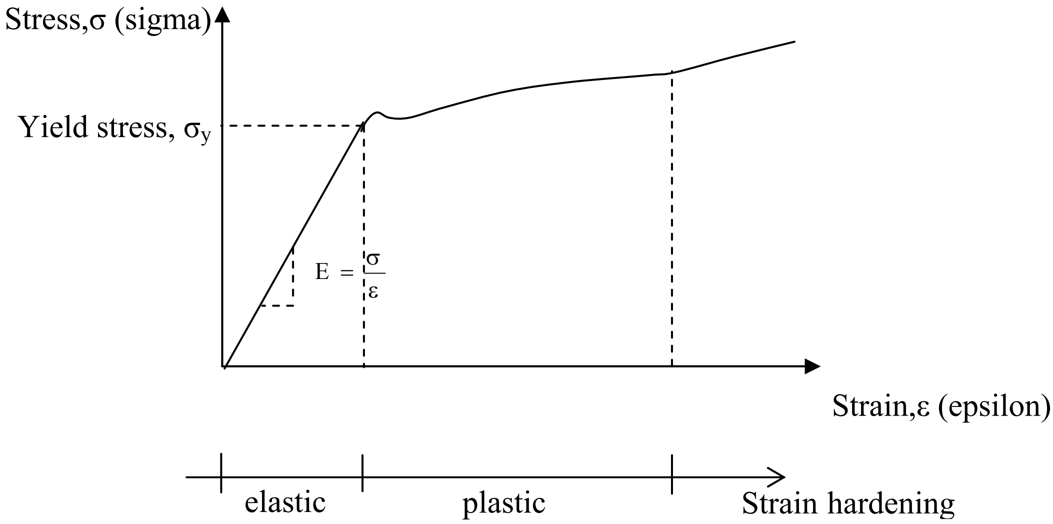


Figure 6.1: The stress strain relationship of steel

- The advantages of plastic behaviour;

  a) Reduce the risk of failure due to the additional load or calculation error.
  b) Give the additional safety to structure.

- The theory of plastic analysis based on;

  a) Stable structure.
  b) Steel in ductile behaviour.
  c) The main point in plastic action is the bending neglecting the influence of any shear force and axial load that may be present.

- Two methods can be used to solve the plastic analysis problem;

  a) Graphical method
  b) Virtual work method

6.2. PLASTIC MOMENT, M_P

- Consider the beam with the cross section (b x h) is subjected to bending, M due to the point load, P.

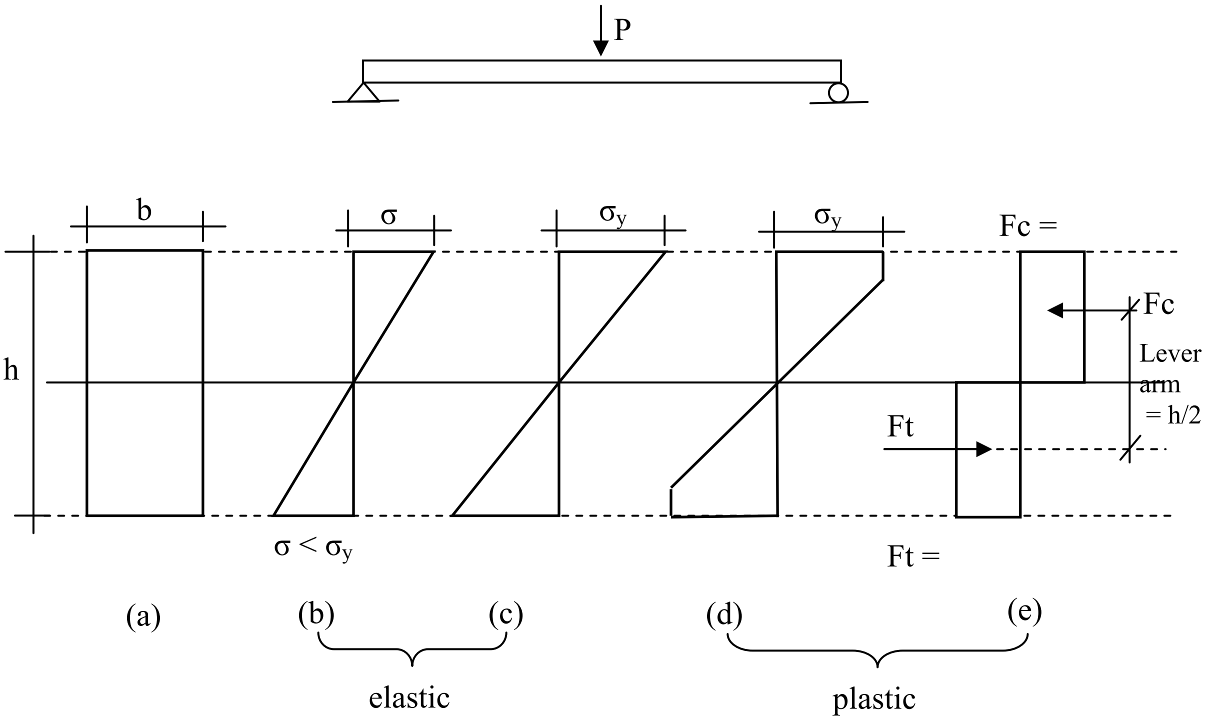


Figure 6.2: Stress distribution diagram

Where;

(a) beam cross section, b x h

(b) stress distribution for entire cross sectional area in the elastic stage.

Bending stress, $\sigma = \dfrac{M.y}{I}$

where M = bending moment
y = bottom or above distance to neutral axis
I = moment of inertia of cross section

(c) outer section achieved the yield stress (σ_y). bending moment due to yield stress called as a yield moment, M_y.

$$\sigma_y = \frac{M_y.y}{I} = \frac{M_y.h/2}{I} = \frac{M_y.h}{2I}$$

$$M_y = \frac{2I\sigma_y}{h} = \frac{2\left(\frac{bh^3}{12}\right).\sigma_y}{h} = \sigma_y\left(\frac{bh^2}{6}\right)$$

$$= \sigma_y.Z$$

$$Z = \text{elastic modulus of the section} = \frac{I}{y}$$

$$\text{for rectangular section, } Z = \frac{bh^2}{6}$$

$$\text{for circle cross section; } Z = \frac{\pi d^3}{32}$$

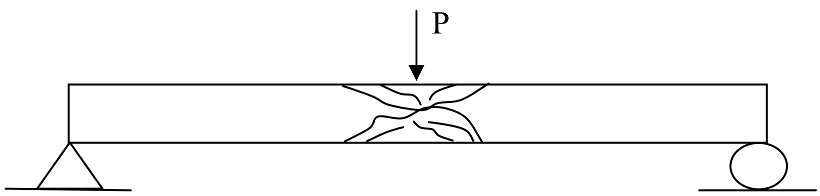
(d) As the load increased further, yielding spreads inwards from the extreme fibers until the cross section become half plastic.

(e) The entire section has become plastic. At this stage, the bending moment known as a plastic moment, M_p.

M_p = load x lever arm
= F x h/2
= $(\sigma_y .bh/2).(h/2)$
= $\sigma_y . bh^2/4$
= $\sigma_y .z_p$

where z_p = plastic modulus of section
for rectangular section, $z_p = bh^2/4$

The plastic hinge is created.



6.3 SHAPE FACTOR, S AND LOAD FACTOR, λ

a) Shape Factor, S

- The ratio of the plastic moment to yield moment. $S = M_p/M_y = Z_p/Z$
- This factor based on cross sectional area and always more than 1.
  - For rectangular section, $S = \dfrac{bh^2/4}{bh^2/6} = 1.5$
  - For circle section,

$$Z_p = \frac{d^3}{3} \quad ; \quad Z = \frac{\pi d^3}{32}$$

$$S = \frac{\left(\dfrac{d^3}{3}\right)}{\left(\dfrac{\pi d^3}{32}\right)} = 1.7 \quad ; \quad \text{where d is the diameter}$$

- For thin-walled section normally takes a value between 1.1 and 1.2.
- For I section the value is 1.15.

b) Load factor, λ

- The ratio of the collapse load to maximum applied load.
- Load factor is based on the cross sectional shapes. Work load depends
  on the value of I and Z while collapse load depends on the cross sectional shapes.
- Consider the rectangular section;

Resistance moment under work load, M;

$$M = \sigma_b \cdot \frac{bh^2}{6}$$

where, σ_b = pemissible stress in bending

$= \sigma_y / 1.5$

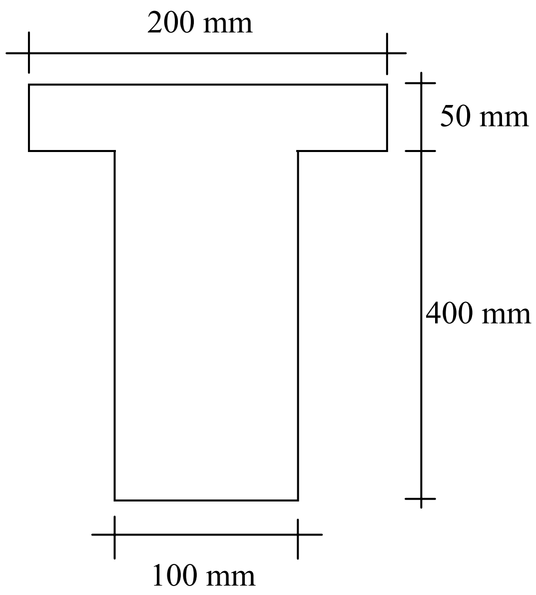
Plastic moment, $Mp = \sigma_y \left(\dfrac{bh^2}{4}\right)$

$$\therefore \lambda = \frac{Mp}{M}$$

$$= 1.5 \text{ x } 1.5$$

$$= 2.25$$

EXAMPLE 6.1



Given, $\sigma_y = 250\ N/mm^2$
Permissible stress, $\sigma_b = 175\ N/mm^2$

Determine :

(1) Elastic modulus,Z
(2) Yield moment, My
(3) Plastic moment, Mp
(4) Plastic modulus,Zp
(5) Shape factor, S
(6) Load factor, λ

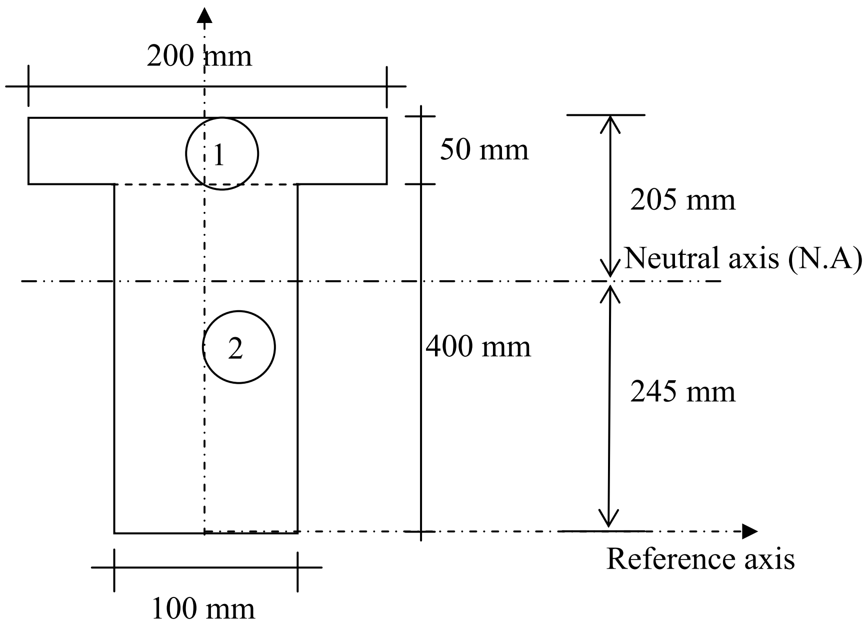
Solution:

Determine the centroid of cross sectional area.

PART	AREA (mm^2)	y (mm)	x (mm)	Ay ($x10^6$)(mm^3)
1	200x50 = 10 000	50/2 + 400 = 425	0	4.25
2	400 x 100 = 40 000	400/2 = 200	0	8
	Σ= 50 000			Σ= 12.25

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{12.25x10^6}{50000} = 245mm$$



Second moment of inertia, I;

PART	**AREA (mm^2)**	$\mathbf{I_x} = \frac{bh^3}{12}$ **(10^6) (mm^4)**	**d (mm)**	**Ad^2 (10^6)(mm^4)**
1	200x50 = 10 000	$\frac{200(50^3)}{12}$ = 25	205-50/2 = 180	324
2	400 x 100 = 40 000	$\frac{100(400^3)}{12}$ =533.33	245-400/2 = 45	81
	Σ= 50 000	Σ=558.33		Σ=405

$$\sum I_{xx} = [Ix + Ad^2]_1 + [Ix + Ad^2]_2$$
$$= 963.33\mathbf{x}10^6 \text{ mm}^4$$

1) Elastic modulus, Z;

$$Z = \frac{I}{y_{max}} = \frac{963.33x10^6}{245} = 3.93x10^6 \text{ mm}^3 \quad \textbf{……….. Ans}$$

2) Yield moment, M_y;

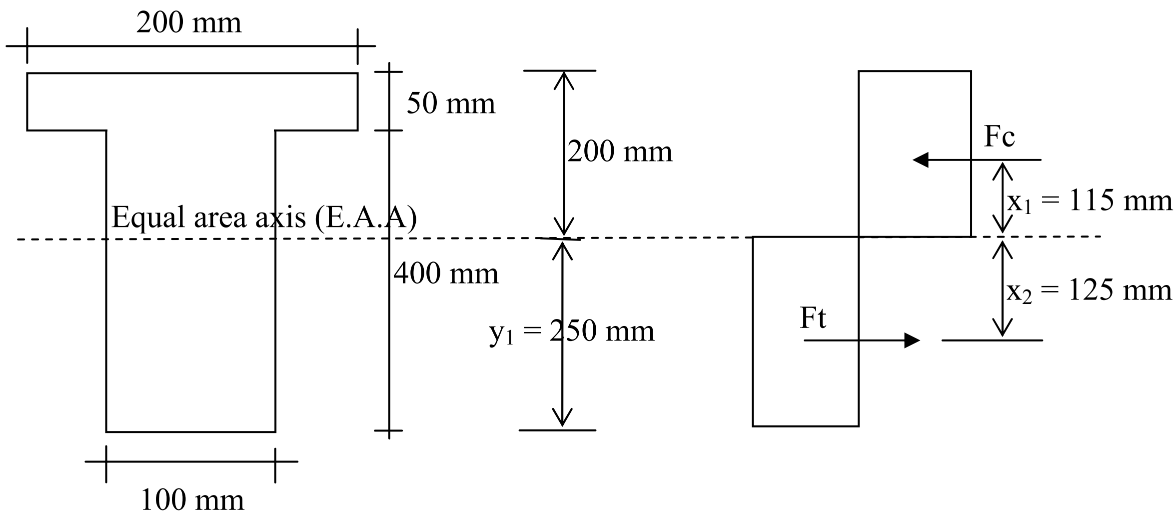
$$M_y = \sigma_y.Z = 250(3.93x10^6) = 982.5\text{MNmm} \textbf{ …….Ans}$$

3) Plastic moment, M_p

Notes:

- Neutral axis ($\sigma = 0$) in the elastic analysis pass through the area of centre.
- Neutral axis in the plastic analysis divide the cross sectional equal area. This is called as the equal area axis (E.A.A).
- For rectangular cross section;

Neutral axis = Equal Area Axis

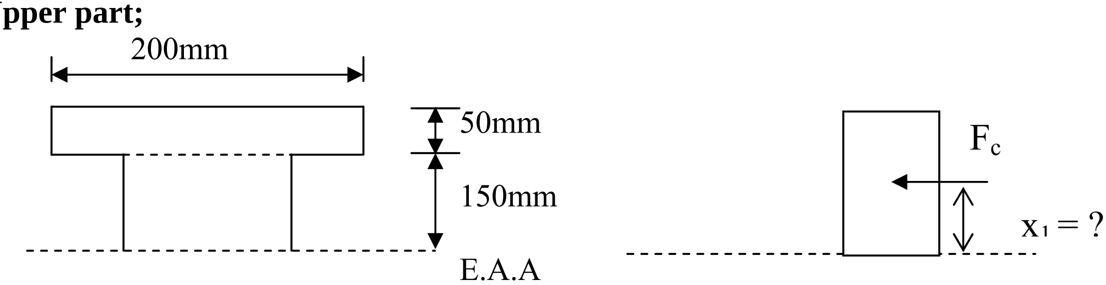


The equal area axis with the distance, y_1 from bottom of section:

$$100(y_1) = 200(50) + 100(400 - y_1)$$

$$y_1 = 250 \text{ mm}$$

Upper part;



1^{st} : Find the centroid of upper part;

PART	AREA (mm^2)	y (mm)	x (mm)	Ay ($\times 10^6$)(mm^3)
1	200x50 = 10 000	150 + 50/2 = 175	0	1.75
2	100x150 = 15 000	150/2 = 75	0	1.125
	Σ= 25 000			Σ= 2.875

2nd : Centroid Fc with the distance, x_1 from equal area axis :

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{2.875x10^6}{25000} = 115mm \quad \ldots\ldots \text{change } \bar{y} \text{ to } x_1$$

Lower part;

3rd : Centroid Ft with the distance, x_2 from equal area axis :

$$x_2 = \frac{y_1}{2} = \frac{250}{2} = 125\,mm$$

$$Mp = \text{force x lever arm} = Ft \text{ x } Z = \sigma_y.A \text{ x } Z$$

$$= 250(100x250)x(125+115)$$

$$= 1.5 \text{ x } 10^9 \text{ Nmm}$$

…….. Ans

4) Plastic modulus, Zp;

$$Z_P = \frac{M_P}{\sigma_y} = \frac{1.5x10^9}{250} = 6x10^6 \text{ mm}^3$$ **………Ans**

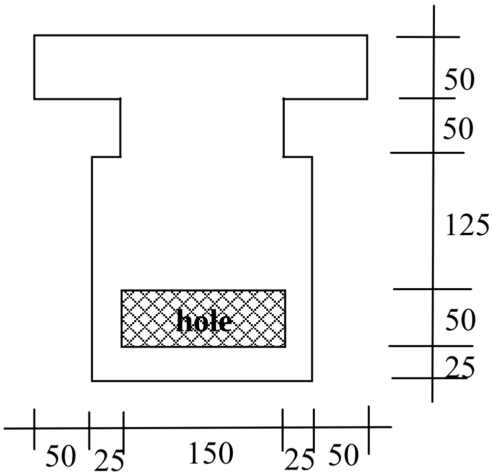
5) Shape factor,S;

$$S = \frac{M_P}{M_Y} = \frac{1.5x10^9}{982.5x10^6} = 1.53$$ **………Ans**

6) Load factor, λ

$$\lambda = \frac{\sigma_y}{\sigma_b} x(S) = \frac{250}{175}(1.53) = 2.19$$ **………Ans**

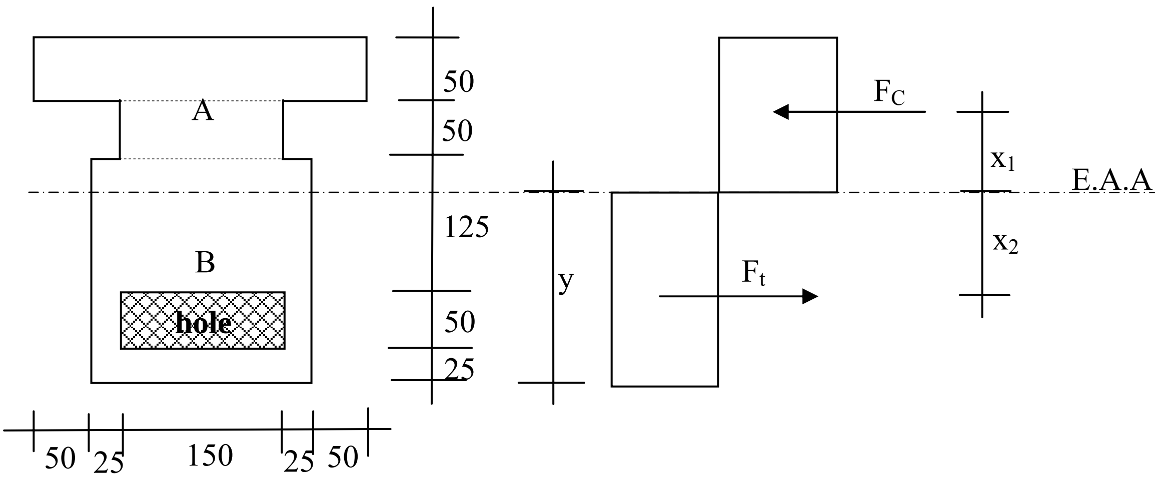
EXAMPLE 6.2



Determine the plastic moment, M_p.
Given : $\sigma_y = 250$ N/mm^2.

All units in mm.

Solution;



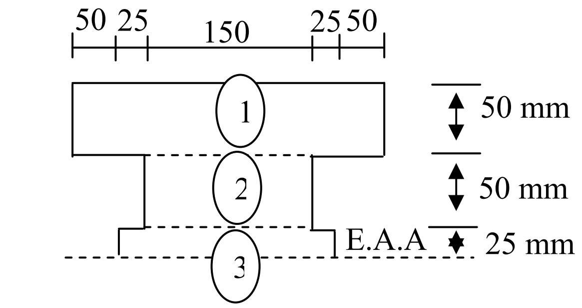
- ✓ Total area = 50(300) + 50(150) + 200(200) – 50(150) = 55,000 mm^2

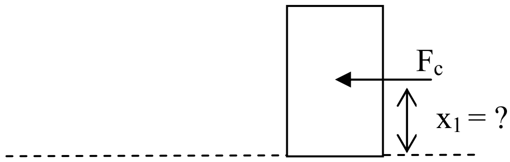
- ✓ Equal area axis located at the distance, y from the bottom of cross sectional area.

$$200(y) - 150(50) = 55000/2$$
$$200y = 27500 - 7500$$
$$y = 175 \text{ mm}$$

- ✓ Lever arm length, $Z = x_1 + x_2$

Upper part;





The centroid of upper part; (the reference axis is located at E.A.A)

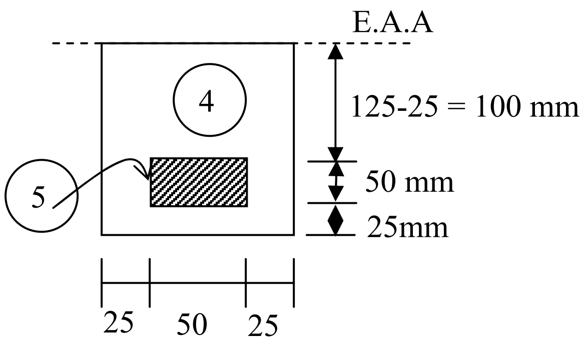
PART	**AREA (mm^2)**	**y (mm)**	**x (mm)**	**Ay (x10^6)(mm^3)**
1	300x50 = 15 000	75 + 50/2 = 100	0	1.5
2	150x50 = 7 500	25 + 50/2 = 50	0	0.375
3	200x 25 = 5 000	25/2 = 12.5	0	0.0625
	Σ= 27 500			Σ= 1.934

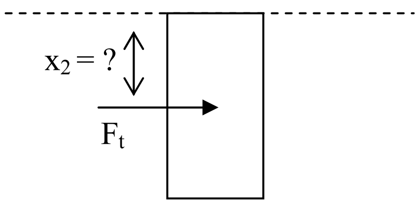
Centroid Fc with the distance, x_1 from equal area axis :

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{1.934x10^6}{27500} = 70.33\text{mm} \quad \ldots\ldots\text{change } \bar{y} \text{ to } x_1$$

Second part;





The centroid of second part; (the reference axis is located at E.A.A)

PART	**AREA (mm^2)**	**y (mm)**	**x (mm)**	**Ay ($x10^6$)(mm^3)**
4	200x175 = 35 000	175/2 = 87.5	0	3.063
5	-150x50 = - 7 500	100 + 50/2 = 125	0	-0.938
	Σ= 27 500			Σ= 2.125

Centroid Ft with the distance, x_2 from equal area axis :

$$\bar{y} = \frac{\sum Ay}{\sum A}$$

$$= \frac{2.125x10^6}{27500} = 77.27mm \quad \ldots\ldots \text{change } \bar{y} \text{ to } x_2$$

$$Mp = \text{force x lever arm} = \text{Ft x Z} = \sigma_y.A \text{ x Z}$$

$$= 250(27500)x(70.33 + 77.27)$$

$$= 1.015 \text{ GNmm}$$

……….. Ans

6.4 THEOREMS IN PLASTIC ANALYSIS

Three following conditions must be satisfied by a structure in its collapse state;

a. The **equilibrium** condition – the system of bending moments must be in equilibrium with the applied load. ($\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$)

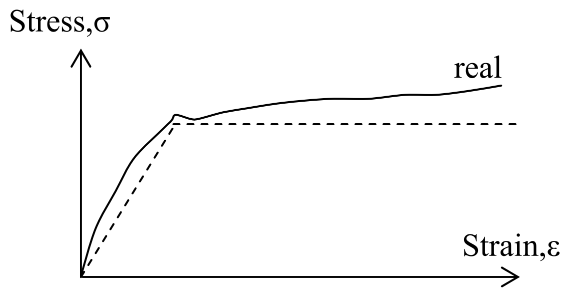
b. The **yield** condition – the bending moment may not exceed the plastic moment, M_p at any point.

c. The **mechanism** condition – sufficient plastic hinges must have formed to reduce all or part of the structure to a mechanism.

6.5 ASSUMPTIONS IN PLASTIC ANALYSIS

a. Obey the Hooke's Law if the moment is not exceed the yield moment, M_y.

b. The relationship of stress-strain must be considered;



c. The cross sectional area are assumed symmetrical about the major axis so that plastic moment takes the same value whether the bending moment is hogging or sagging.

d. A cross section is assumed elastic and rigid until the plastic moment is reached at which point a hinge forms and the rotational stiffness becomes zero.

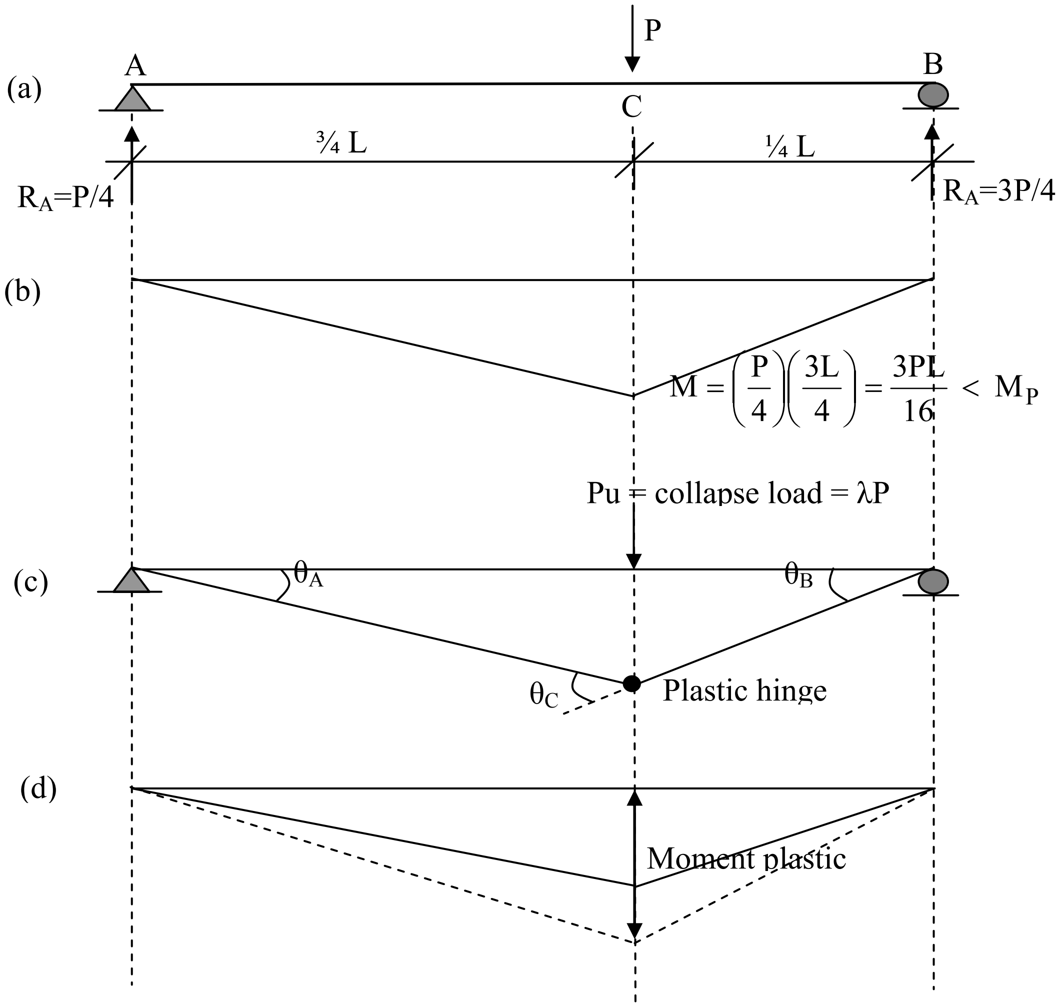
e. Neglecting the influence of any axial load or shear force that may be presented.

$$\sigma_{bending} >>> \sigma_{axial}$$

$$\sigma_{bending} = \frac{My}{I} >>> \sigma_{axial} = \frac{P}{A}$$

6.6 COLLAPSE LOAD AND COLLAPSE MECHANISM

Consider the statically determinate structure is simply supported with point load, P;



Notes:

(a) The beam is subjected to work load, P in elastic condition.

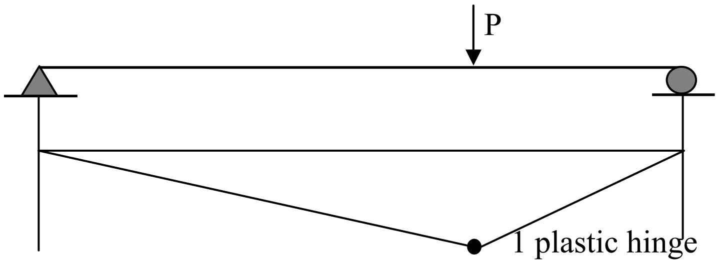
(b) Bending moment diagram (BML) ($M_{max} < M_P$ because its still in elastic condition)

(c) As the load increased further, the plastic hinge is created at C. At this stage, the beam achieved the collapse load, Pu or λP and bending moment at C known as plastic moment, M_P.

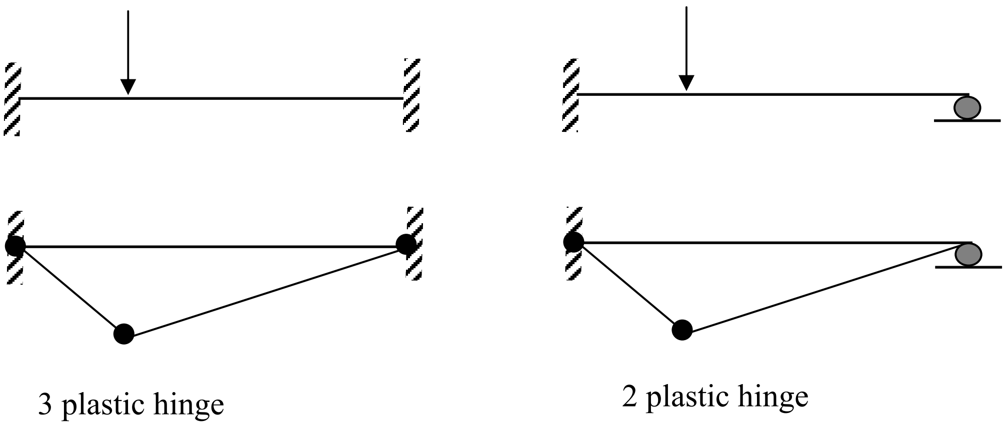
(d) Then, the beam will fail and can be as a mechanism. Mechanism is an assemblage of members or parts which undergoes large displacement due to even a very small load. Collapse load or ultimate load, Pu/λP with λ is a load factor which is the ratio of the collapse load to work load.

6.7 THE LOCATIONS OF PLASTIC HINGE

- For statically determinate beam, only one hinge is needed to ensure the beam is in mechanism condition.



- For statically indeterminate beam, at least 2 hinges are needed to ensure the beam is in mechanism condition.

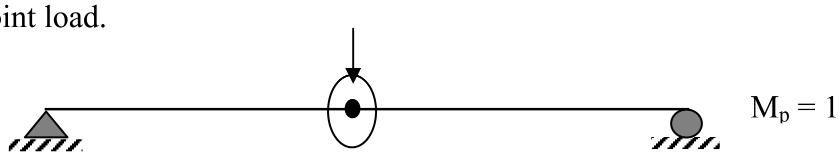


- Generally, there are two types of hinge in the beam structure. Normal hinge is located at the pinned ended or roller ended support and no moment. But for the continuous beam, the plastic hinge is located at the continuous support. Plastic hinge have the moment plastic, M_P and existed at the following locations.

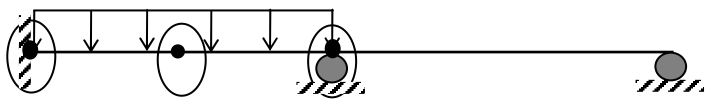
- The plastic hinge can be found in these locations;

1. At fixed ended support.

2. At point load.

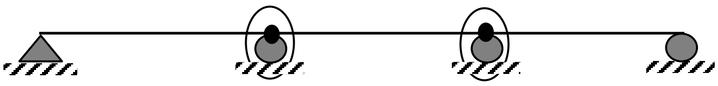


3. At the centre of uniform distributed load.



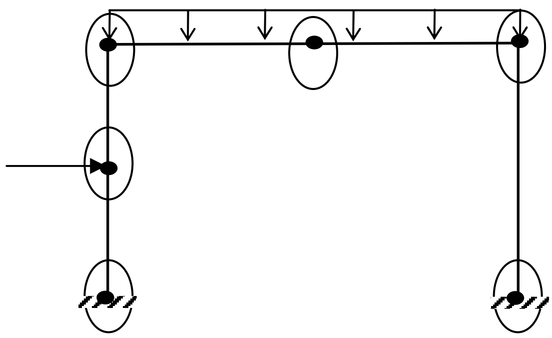
$M_p = 3$

4. At continuous support.



$M_p = 2$

5. At joint of portal frame.



$M_p = 6$

- The additional prediction in plastic analysis are;

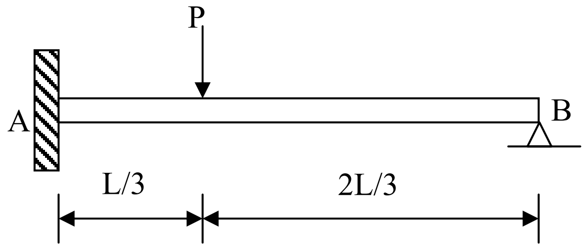
  a) At joint of the structure which is have two members. The plastic hinge is created at the weakest member or at the smallest cross sectional area.

  b) At the joint which is having more than three members, the plastic hinge is created in the member itself even the cross sectional area is bigger than other members.

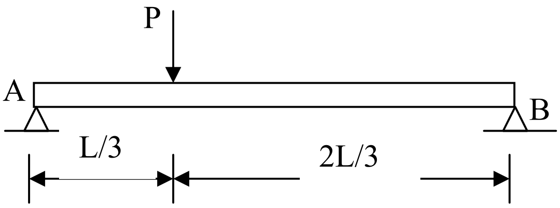
6.8 PLASTIC ANALYSIS BEAM USING THE GRAPHICAL METHOD

EXAMPLE 6.3

Determine plastic moment, M_p and the value of P required to cause collapse using graphical method.



Solution;



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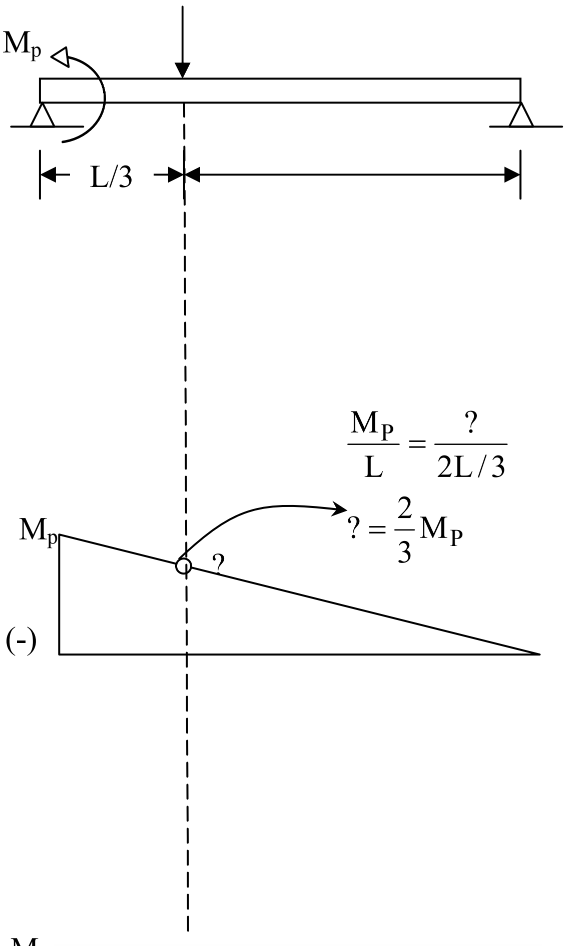


$$V_A + V_B = P$$

$$+M_A = 0$$

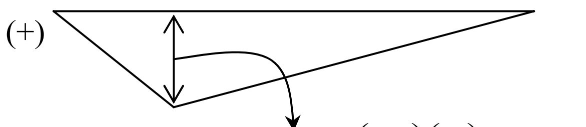
$$P\left(\frac{L}{3}\right) - V_B(L) = 0$$

$$V_B = \frac{P}{3}, V_A = \frac{2P}{3}$$

$$\frac{M_P}{L} = \frac{?}{2L/3}$$

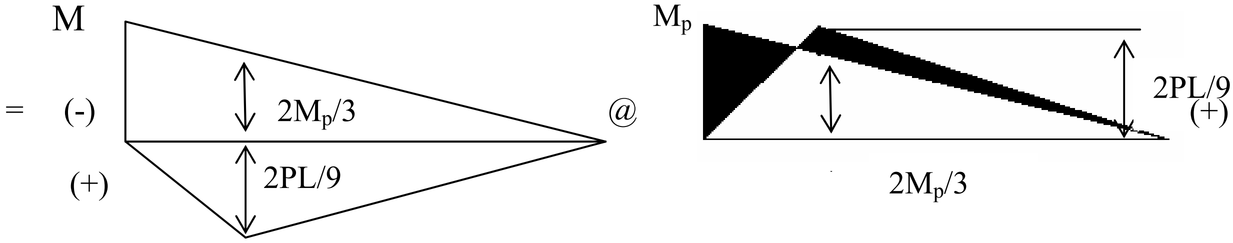
$$? = \frac{2}{3}M_P$$

Draw BMD;



$$M_{max} = \left(\frac{2P}{3}\right)\left(\frac{L}{3}\right) = \frac{2PL}{9}$$

Moment combination diagram;



From the moment combination diagram;

$$-M_P - \frac{2M_P}{3} + \frac{2PL}{9} = 0$$

Plastic moment, M_p → $-M_P - \frac{2M_P}{3} + \frac{2PL}{9} = 0$

$$-\frac{5M_P}{3} = -\frac{2PL}{9}$$

$$M_P = \left(\frac{2PL}{9}\right)\frac{3}{5} = \frac{6PL}{45}$$

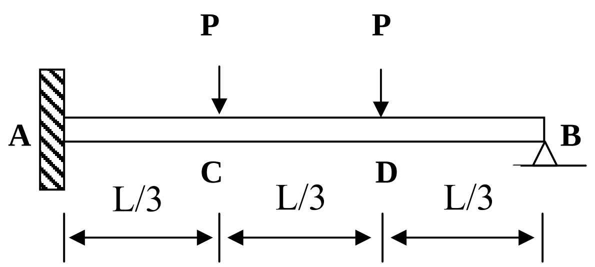
Value of P → $-M_P - \frac{2M_P}{3} + \frac{2PL}{9} = 0$

$$-\frac{5M_P}{3} = -\frac{2PL}{9}$$

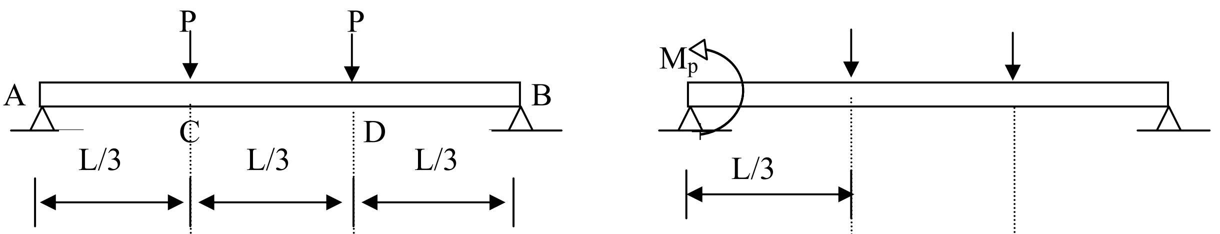
$$P = \left(\frac{5M_P}{3}\right)\frac{9}{2L} = \frac{45M_P}{6L}$$

EXAMPLE 6.4

Determine the value of P required causing collapse using graphical method.



Solution;

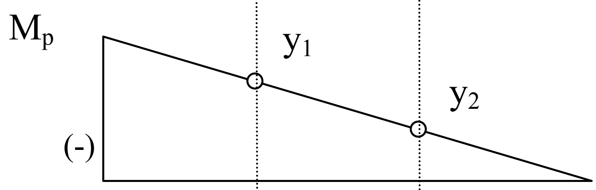


$$V_A + V_B = 2P$$

$$\circlearrowright + M_A = 0$$

$$P\left(\frac{L}{3}\right) + P\left(\frac{2L}{3}\right) - V_B(L) = 0$$

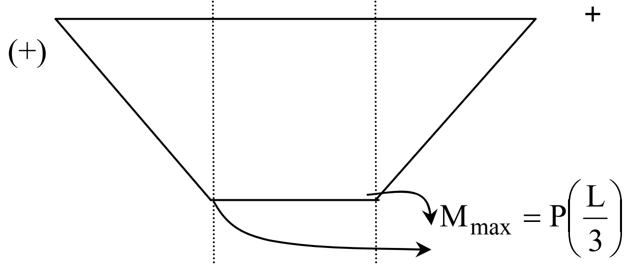
$$V_B = P, V_A = P$$



$$\frac{M_P}{L} = \frac{y_1}{2L/3} \qquad \frac{M_P}{L} = \frac{y_2}{L/3}$$

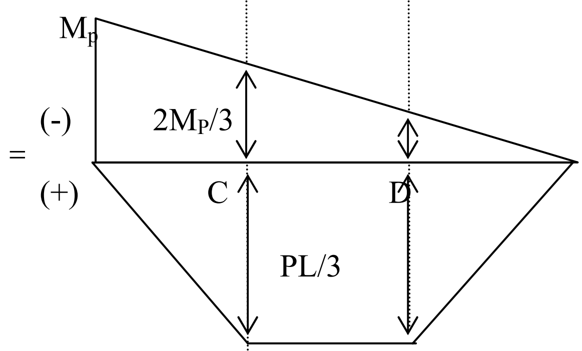
$$y_1 = \frac{2}{3}M_P \qquad y_2 = \frac{1}{3}M_P$$

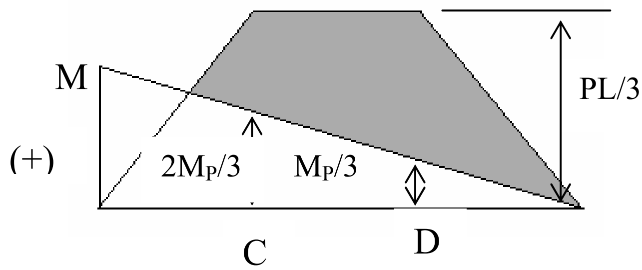
Draw BMD;



$$M_{max} = P\left(\frac{L}{3}\right)$$

Moment combination diagram;





From the moment combination diagram;

At point C:

$$-M_P - \frac{2M_P}{3} + \frac{PL}{3} = 0$$

The value of P;

$$P = 5M_P / L$$

At point D:

$$-M_P - \frac{M_P}{3} + \frac{PL}{3} = 0$$

The value of P;

$$P = 4M_P / L$$

6.9 Plastic Analysis Beam Using The Virtual Work Method

- ✓ The principal of virtual work;

  External Virtual Work (EVW) = Internal Virtual Work(IVW)

  (subjected to external/applied load) (subjected to plastic hinge)

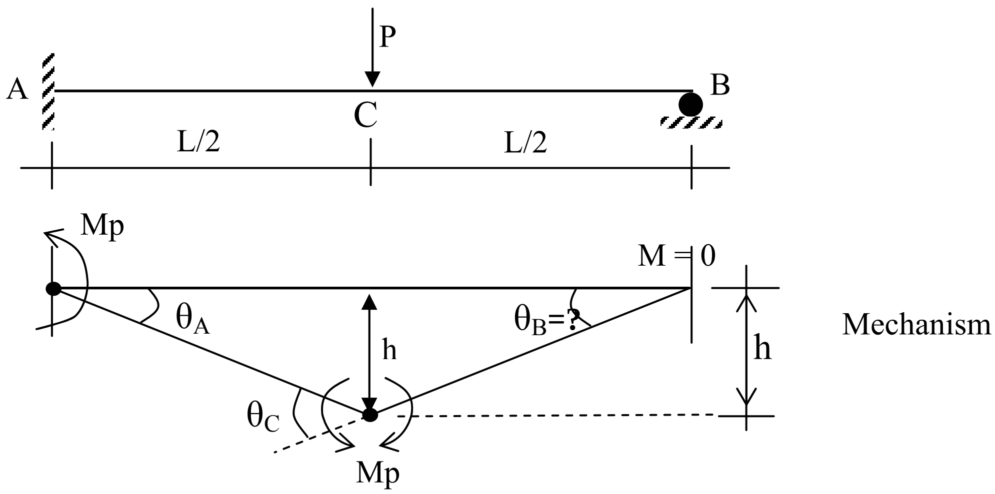
- ✓ If more than one applied load acted on the beam, the mode failure should be considered.

- ✓ Consider the beam subjected to concentrated load, P;

6.9.1 External Work For The Beam Undergoes The Point Load, P.

<u>EXAMPLE 6.5</u>

Determine plastic moment, M_p and the value of P required to cause collapse using the virtual work method.



Assumes tan $\theta_A = \theta$ (tangent is neglected because of the small value)

$$\theta_A = h/\left(\frac{L}{2}\right) = \theta\ ,\ h = \left(\frac{L}{2}\right)\theta$$

$$\theta_B = h/\left(\frac{L}{2}\right) = \left(\frac{L}{2}\right)\theta/\left(\frac{L}{2}\right) = \theta$$

$$\theta_C = \theta_A + \theta_B = 2\theta$$

External work = Internal work

$$P(h) = Mp(\theta_A) + Mp(\theta_C)$$

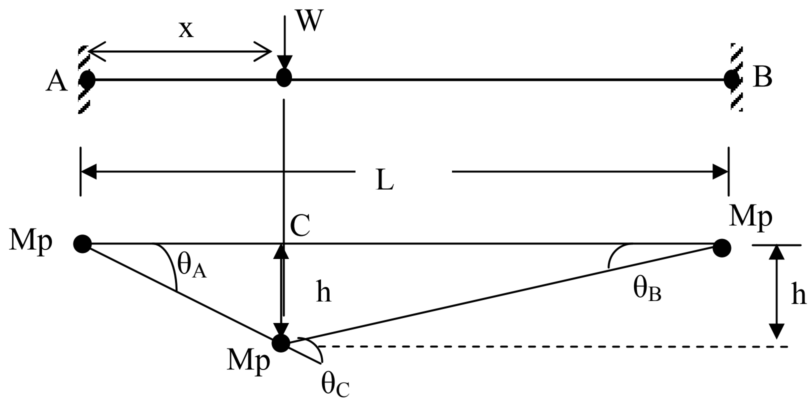
$$P\left(\frac{L}{2}\right)\theta = Mp(\theta) + Mp(2\theta)$$

$$P\left(\frac{L}{2}\right)\theta = 3Mp(\theta)$$

$$P = \frac{6Mp}{L} \quad \text{or} \quad Mp = \frac{PL}{6}$$

EXAMPLE 6.6

The beam is fixed at both support and subjected to point load, W with distance, x from A. Determine plastic moment, M_p in x using the virtual work method.



Assumes tan $\theta_A = \theta$ (tangent is neglected because of the small value)

$$\theta_A = \frac{h}{x} = \theta\ ,\ h = x\theta$$

$$\theta_B = \frac{h}{(L-x)} = \frac{x\theta}{(L-x)}$$

$$\theta_C = \theta_A + \theta_B = \theta + \frac{x\theta}{(L-x)} = \frac{\theta L}{(L-x)}$$

External work = Internal work

$$W(h) = Mp(\theta_A) + Mp(\theta_B) + Mp(\theta_C)$$

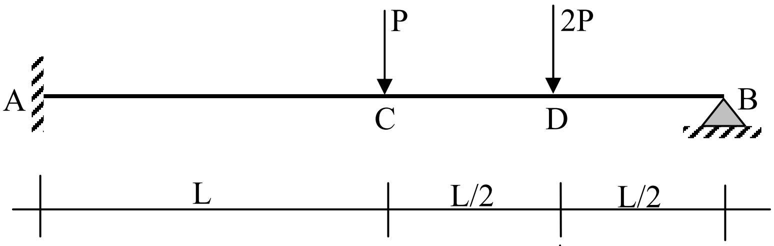
$$W(x\theta) = Mp(\theta) + Mp\left[\frac{x\theta}{(L-x)}\right] + Mp\left[\frac{\theta L}{(L-x)}\right]$$

$$(Wx)\theta = M_p\left[\frac{2L}{(L-x)}\right]\theta$$

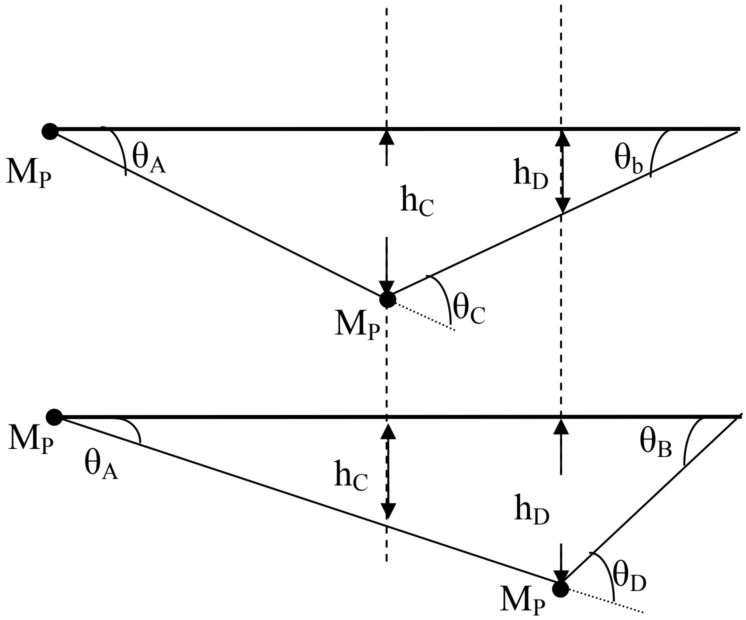
$$W = \frac{2L}{(L-x)x}M_P \quad \text{or} \quad M_p = Wx\frac{(L-x)}{2L}$$

EXAMPLE 6.7

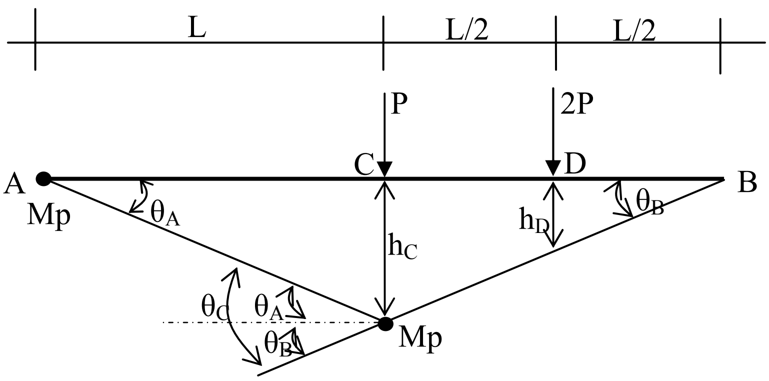
Determine the maximum P at the propped beam using the virtual work method.



Solution;



Mode failure 1 – Beam mechanism AB;



Assumes tan $\theta_A = \theta$ (tangent is neglected because of the small value)

$$\theta_A = \left(\frac{h_C}{L}\right) = \theta \; ; \; h_C = L\theta$$

$$\theta_B = \left(\frac{h_C}{L}\right) = \left(\frac{L\theta}{L}\right) = \theta$$

$$h_D = \theta_B\left(\frac{L}{2}\right) = \frac{L\theta}{2}$$

$$\theta_C = \theta_A + \theta_B = 2\theta$$

External work = **Internal work**

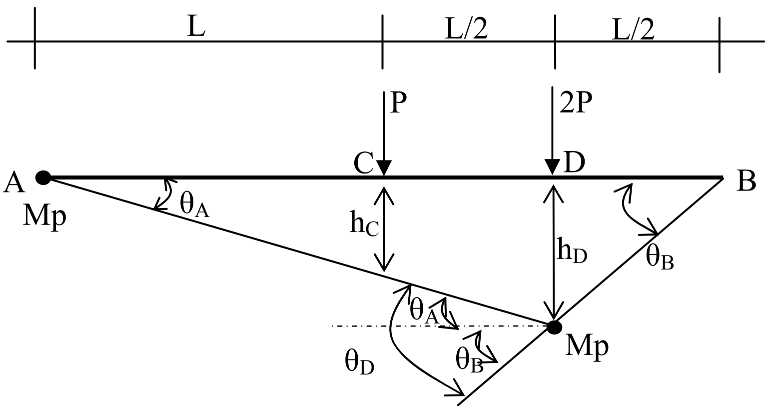
$$P\,(h_C) + 2P\,(h_D) = Mp(\theta_A) + Mp(\theta_C)$$

$$P\,(L\theta) + 2P\left(\frac{L\theta}{2}\right) = Mp(\theta) + Mp(2\theta)$$

$$2PL = 3Mp(\theta)$$

$$P = \frac{3Mp}{2L}$$

Mode failure 2 – Beam mechanism AB;



Assumes tan $\theta_A = \theta$

$$\theta_A = \left(\frac{h_D}{3L/2}\right) = \theta \; ; \; h_D = \frac{3L}{2}\theta$$

$$\theta_B = \left(\frac{h_D}{L/2}\right) = \left(\frac{L\theta}{L}\right) = \theta$$

$$h_D = \theta_B\left(\frac{L}{2}\right) = \frac{L\theta}{2}$$

$$\theta_C = \theta_A + \theta_B = 2\theta$$

External work = Internal work

$$P(h_C) + 2P(h_D) = Mp(\theta_A) + Mp(\theta_C)$$

$$P(L\theta) + 2P\left(\frac{L\theta}{2}\right) = Mp(\theta) + Mp(2\theta)$$

$$2PL = 3Mp(\theta)$$

$$P = \frac{3Mp}{2L}$$

* Maximum Load, $P_u = \dfrac{3M_P}{2L}$

EXERCISE 6.1:

1. Determine the value of P required causing collapse using the virtual work method in Figure 6.1.

2. The simply supported beam of length 6m and plastic moment 150kNm carries a concentrated load P at midspan in Figure 6.2. Find the value of P required to cause collapse. Use the virtual work method.

3. A continuous beam comprises three equal spans of length, L and has a uniform section with plastic moment, M_p. It carries point loads of P at the centre of the two outer spans and 2P at the centre of the middle span in Figure 6.3. Determine the value of P required to cause collapse. Use the virtual work method.

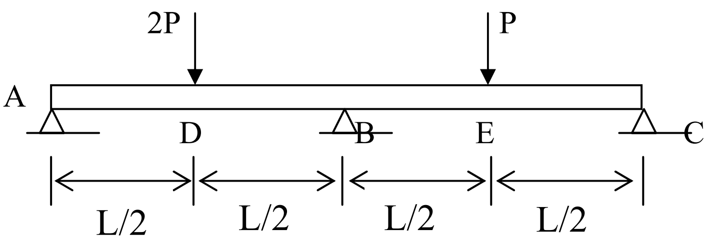


Figure 6.1

[Ans: $P_{\text{at D}} = \dfrac{3M_P}{L}$, $P_{\text{at E}} = \dfrac{6M_P}{L}$ **]**

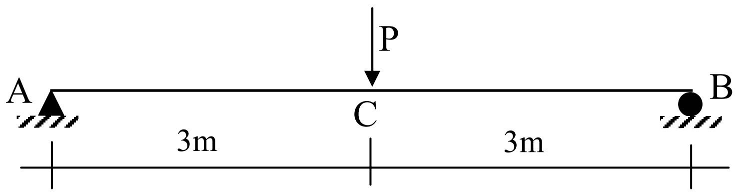


Figure 6.2

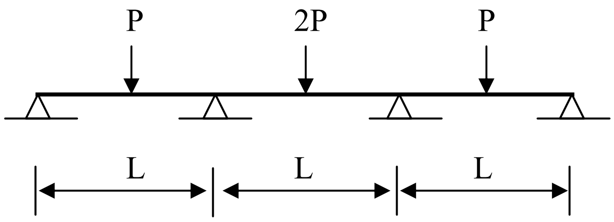
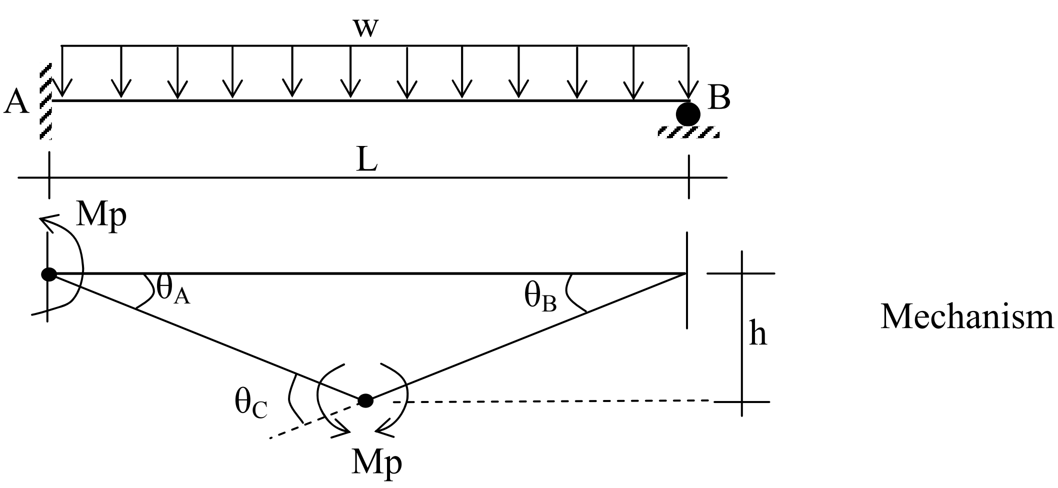


Figure 6.3

6.9.2 External Work For The Beam Undergoes The Uniform Load, q.

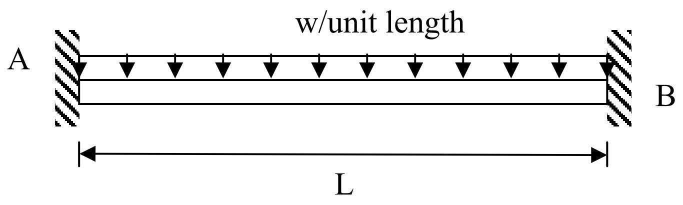
- External work for the beam undergoes the uniform distributed load, q :

External work = w x area of mechanism along the load
= w x (1/2 x L x h)

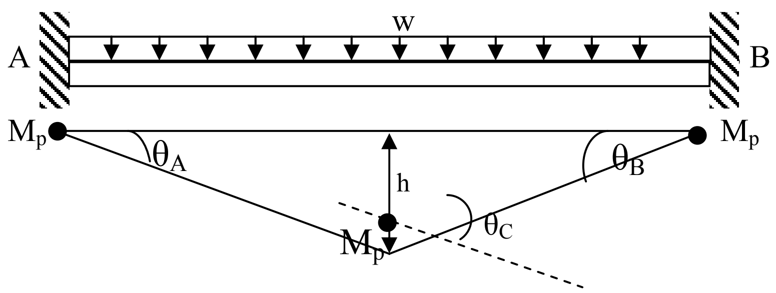


EXAMPLE 6.8

The beam is fixed at both support and subjected to uniform distributed load, w. Determine plastic moment, M_p using the virtual work method.



Solution;
The uniform distributed load can be changed to point load and it is acted at the centre of the span.



Assumes $\tan\theta_A = \theta$

$$\theta_A = \left(\frac{h}{L/2}\right) = \theta\ ;\quad h = \frac{L}{2}\theta$$

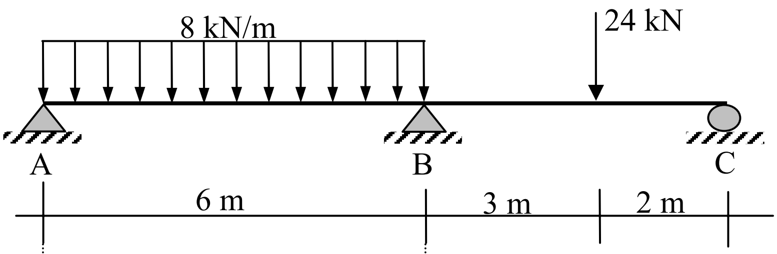
$$\theta_B = \left(\frac{h}{L/2}\right) = \left(\frac{L\theta}{L}\right) = \theta$$

$$\theta_C = \theta_A + \theta_B = 2\theta$$

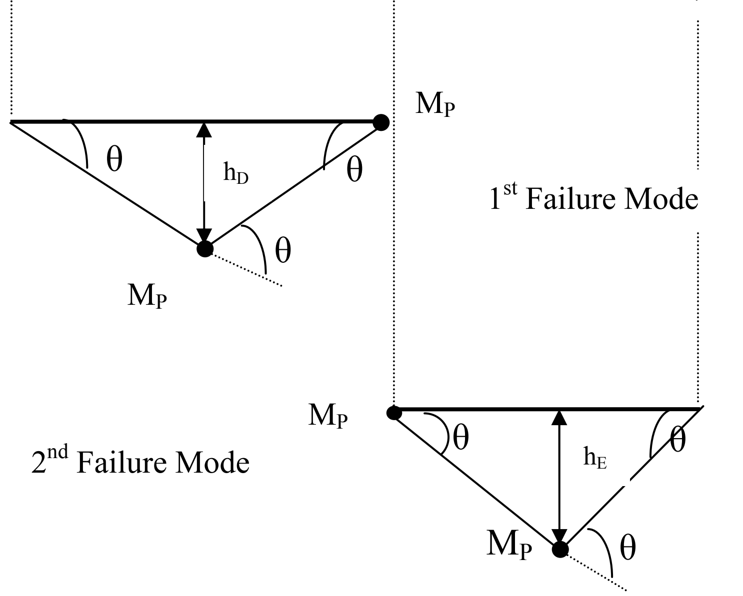
External work	=	Internal work
(1/2 x h x L)(w)	=	$Mp(\theta_A) + Mp(\theta_B) + Mp(\theta_C)$
$\frac{1}{2} \times \frac{L}{2}\theta \times L \times w$	=	$Mp(\theta) + Mp(\theta) + Mp(2\theta)$
$\frac{wL^2}{4}\theta$	=	$4Mp(\theta)$
Mp	=	$\frac{wL^2}{16}$

EXAMPLE 6.9

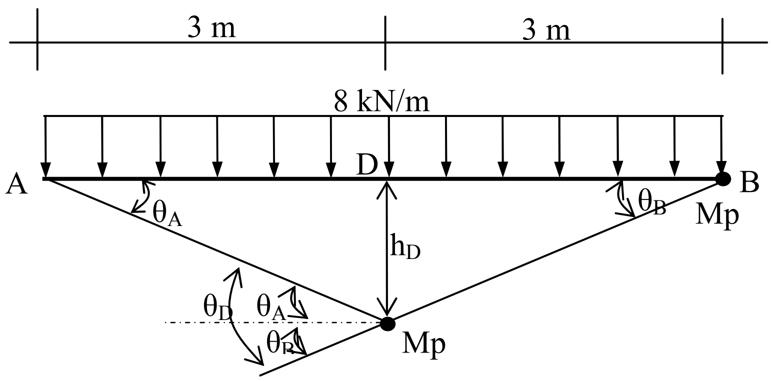
Determine the maximum of plastic moment for this beam.



Solution;



Mode failure 1 – Beam mechanism AB;



Assumes tan $\theta_A = \theta$

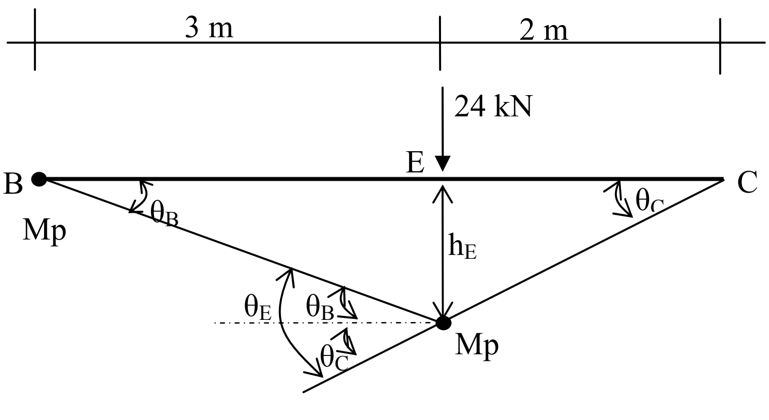
$\theta_A = \left(\frac{h_D}{3}\right) = \theta \; ; \; h_D = 3\theta$

$\theta_B = \left(\frac{h_D}{3}\right) = \left(\frac{3\theta}{3}\right) = \theta$

$\theta_D = \theta_A + \theta_B = 2\theta$

External work	=	Internal work
$(1/2 \text{ x } h_D \text{ x L})(w)$	=	$Mp(\theta_B) + Mp(\theta_D)$
$\frac{1}{2}$ x 3θ x 6 x 8	=	$Mp(\theta) + Mp(2\theta)$
72 θ	=	$3Mp(\theta)$
Mp	**=**	**24 kNm**

Mode failure 2 – Beam mechanism BC;



Assumes tan $\theta_B = \theta$

$\theta_B = \left(\frac{h_E}{3}\right) = \theta \; ; \; h_E = 3\theta$

$\theta_C = \left(\frac{h_E}{2}\right) = \left(\frac{3\theta}{2}\right) = 1.5\theta$

$\theta_E = \theta_B + \theta_C = \theta + 1.5\theta = 2.5\theta$

External work	**=**	**Internal work**

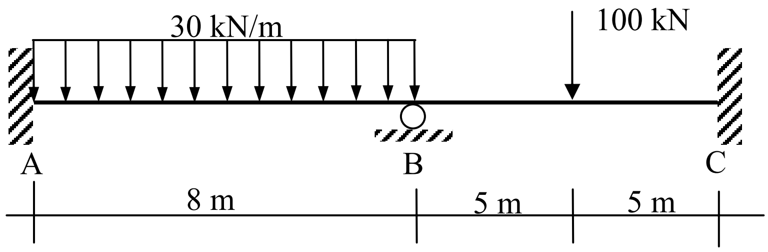
$$(P \times h_E) = Mp(\theta_B) + Mp(\theta_E)$$
$$24 \times 3\theta = Mp(\theta) + Mp(2.5\theta)$$
$$72\,\theta = 3.5Mp(\theta)$$

$$\mathbf{Mp = 20.6\ kNm}$$

From two mode failure, indicated that the maximum of plastic moment; $M_p = 24$ kNm

EXERCISE 6.1:

1. Determine the failure mode and plastic moment that might be occurred in the beam.



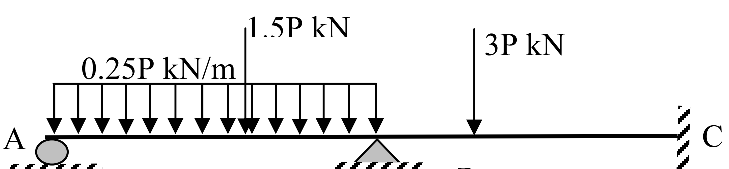
[Ans: $M_{p(AB)}$ = 120kNm, $M_{p(BC)}$ = 16.67kNm]

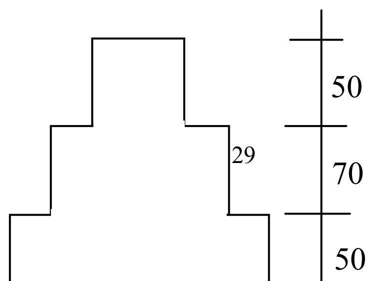
EXAMPLE 6.10

The beam is subjected to the uniform distributed load and point load as shown in the figure. Given the unit of cross section is mm.

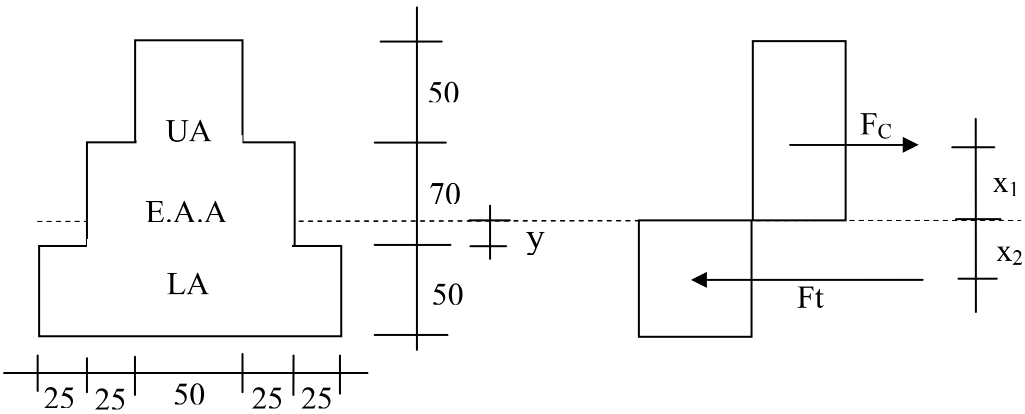
a) Determine the uniform distributed load that can be acted in the beam. Given, $\sigma_y = 250$ N/mm^2.

b) Find the value of P required to cause collapse using the virtual work method.





Solution;



✓ The equal area axis (E.A.A) is located at the distance, y as shown in the figure.

$$\text{Upper area (UA)} = \text{Lower area (LA)}$$
$$50(50) + 100(70\text{-}y) = 100(y) + 50(150)$$
$$200y = 2000$$
$$y = 10 \text{ mm}$$

✓ Lever arm length, $Z = x_1 + x_2$

- In the upper area, the centroid distance, x_1:

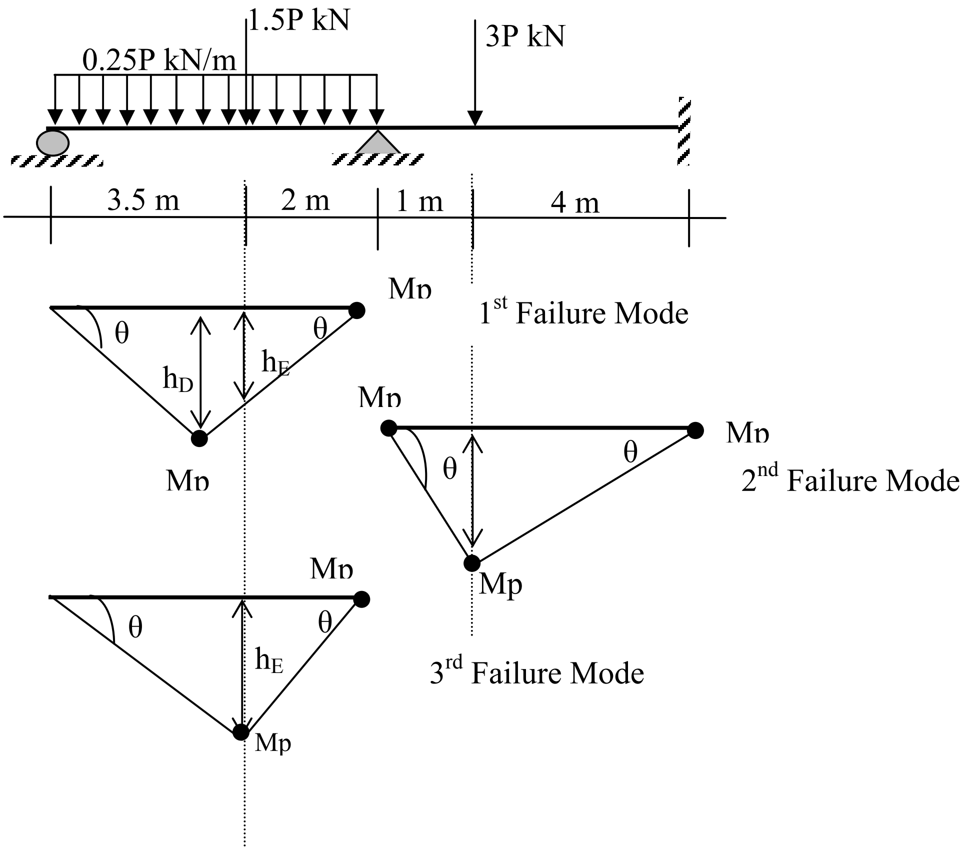
$$x_1 = \frac{50(50)(85) + 100(60)(30)}{50(50) + 100(60)} = 46.18\,\text{mm}$$

- In the lower area, the centroid distance, x_2:

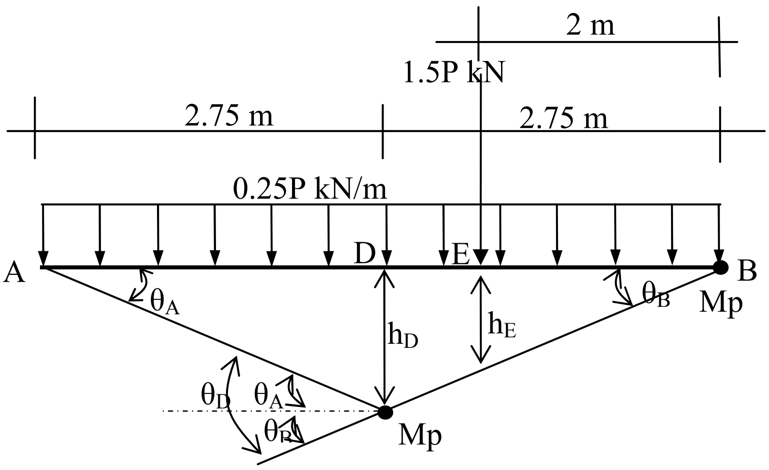
$$x_2 = \frac{150(50)(35) + 100(10)(5)}{150(50) + 100(10)} = 31.47\,\text{mm}$$

✓ Plastic moment, $Mp = \text{force x lever arm} = Ft \text{ x } Z$

$= \sigma_y .A .Z = 250(8500)(46.18+31.47) = 165$ kNm **.... Ans**



Mode failure 1 – Beam mechanism AB;



Assumes tan $\theta_A = \theta$ (tangent is neglected because of the small value)

$\theta_A = \left(\frac{h_D}{2.75}\right) = \theta$; $h_D = 2.75\theta$

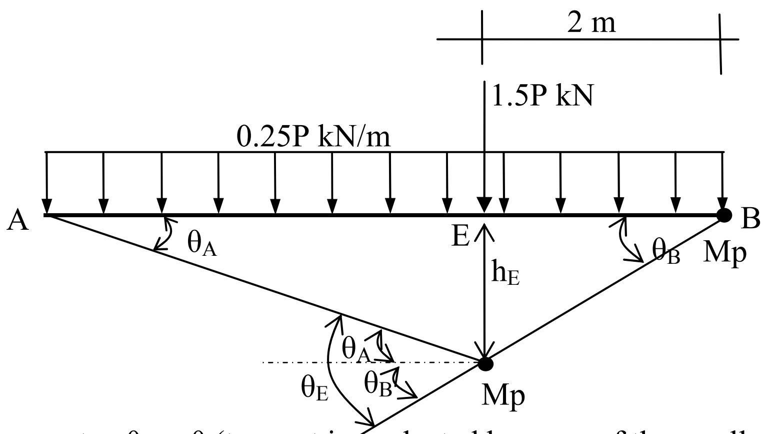
$\theta_B = \left(\frac{h_D}{2.75}\right) = \left(\frac{2.75\theta}{2.75}\right) = \theta$

$h_E = \theta_B(2) = 2\theta$

$\theta_D = \theta_A + \theta_B = \theta + \theta = 2\theta$

External work	=	Internal work
$(1/2 \times L \times h_D)(0.25P) + 1.5P(h_E)$	=	$Mp(\theta_B) + Mp(\theta_D)$
$(1/2 \times 5.5 \times 2.75\theta)(0.25P) + 1.5P(2\theta)$	=	$Mp(\theta) + Mp(2\theta)$
$4.89P(\theta)$	=	$3Mp(\theta)$
P	=	**0.613 Mp**

Mode failure 2: – Beam mechanism AB;



Assumes tan $\theta_A = \theta$ (tangent is neglected because of the small value)

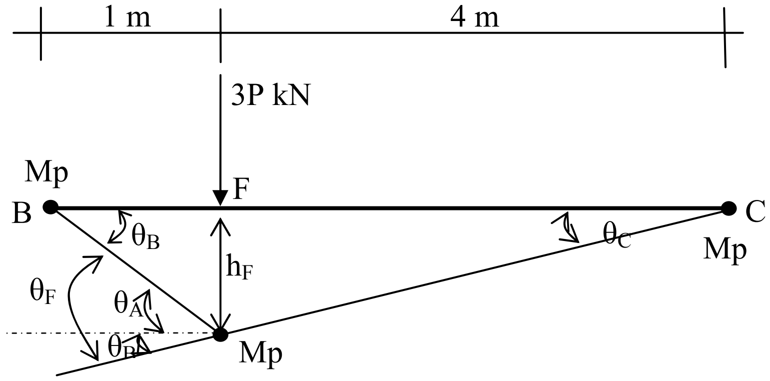
$\theta_A = \left(\frac{h_E}{3.5}\right) = \theta$; $h_E = 3.5\theta$

$\theta_B = \left(\frac{h_E}{2}\right) = \left(\frac{3.5\theta}{2}\right) = 1.75\theta$

$\theta_E = \theta_A + \theta_B = \theta + 1.75\theta = 2.75\theta$

External work	=	Internal work
$(1/2 \times L \times h_E)(0.25P) + 1.5P(h_E)$	=	$Mp(\theta_B) + Mp(\theta_E)$
$(1/2 \times 5.5 \times 3.5\theta)(0.25P) + 1.5P(3.5\theta)$	=	$Mp(1.75\theta)$ + $Mp(2.75\theta)$
$7.66P(\theta)$	=	$4.5Mp(\theta)$
P	=	**0.587 Mp**

Mode failure 3: – Beam mechanism BC;



Assumes tan $\theta_B = \theta$ (tangent is neglected because of the small value)

$$\theta_B = \left(\frac{h_F}{1}\right) = \theta\ ;\ h_F = \theta$$

$$\theta_C = \left(\frac{h_F}{4}\right) = \left(\frac{\theta}{4}\right) = 0.25\theta$$

$$\theta_F = \theta_B + \theta_C = \theta + 0.25\theta = 1.25\theta$$

External work	**=**	**Internal work**
$3P\ (h_F)$	=	$Mp(\theta_B) + Mp(\theta_C) + Mp(\theta_F)$
$3P(\theta)$	=	$Mp(\theta) + Mp(0.25\theta) + Mp(1.25\theta)$
$3P(\theta)$	=	$2.5Mp(\theta)$
P	**=**	**0.833 Mp**

From three mode failures, the maximum load occurred at 3rd Failure Mode;

$P = 0.58M_P$

$= 0.58\text{x}165 = 95.7$ kN **…….Ans**

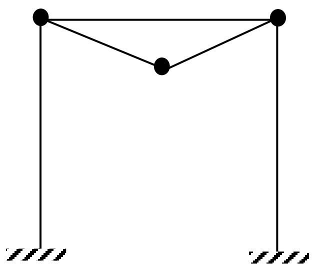
q $= 0.25(95.7) = 23.9$ kN/m **…….Ans**

6.10 Plastic Analysis Frame Using the Virtual Work Method

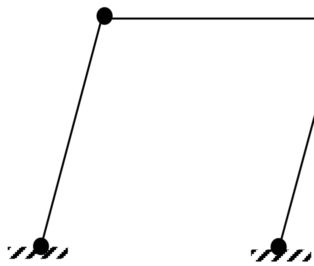
- Two categories of mechanism in portal frame;
  a) Free mechanism
  b) Combined mechanism

- For plastic analysis of frames, there are three types of mechanism;

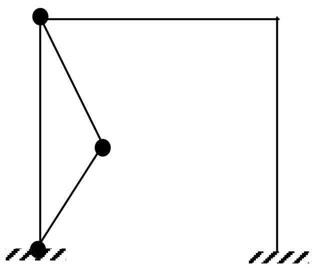
(a) Beam mechanism



(b) Sway mechanism



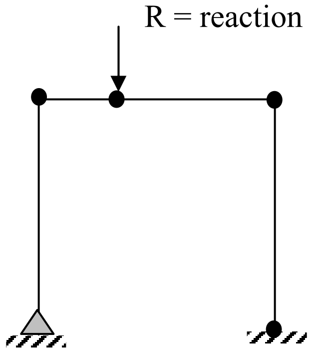
(c) Mekanisme g



- Generally, the number of free mechanism (Me) can be determined by using formula;

Me = N – d

Where N = position number of plastic hinge that might be occurred
d = number of redundant (d = R – 3)
R = reaction occurred



R = 5, N = 4
d = 5 – 3 = 2

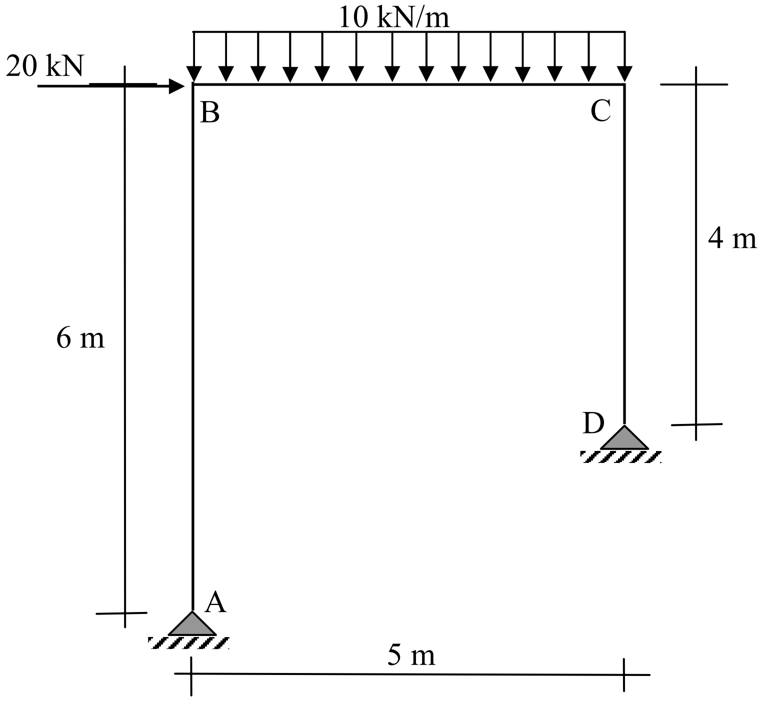
Me = 4 – 2 = 2 (sway + beam)

Total of Mechanism = 2 + 1 = 3

Combine mechanism

EXAMPLE 6.11

Determine the maximum moment plastic from the frame shown below.

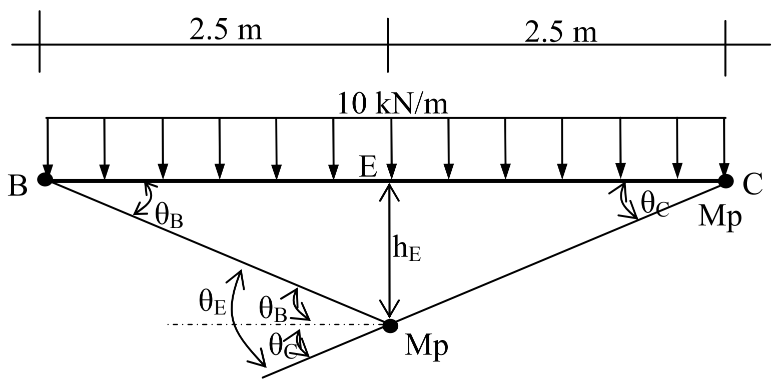


Solution;

$$Me = N - d \qquad (R = 4 , N = 3 \;;\; d = 4 - 3 = 1)$$
$$= 3 - 1 = 2$$

Total of Mechanism = 2 + 1 = 3

Mode failure 1 – Beam mechanism BC; plastic hinge occurred at B, C and E.



Assumes tan $\theta_B = \theta$ (tangent is neglected because of the small value)

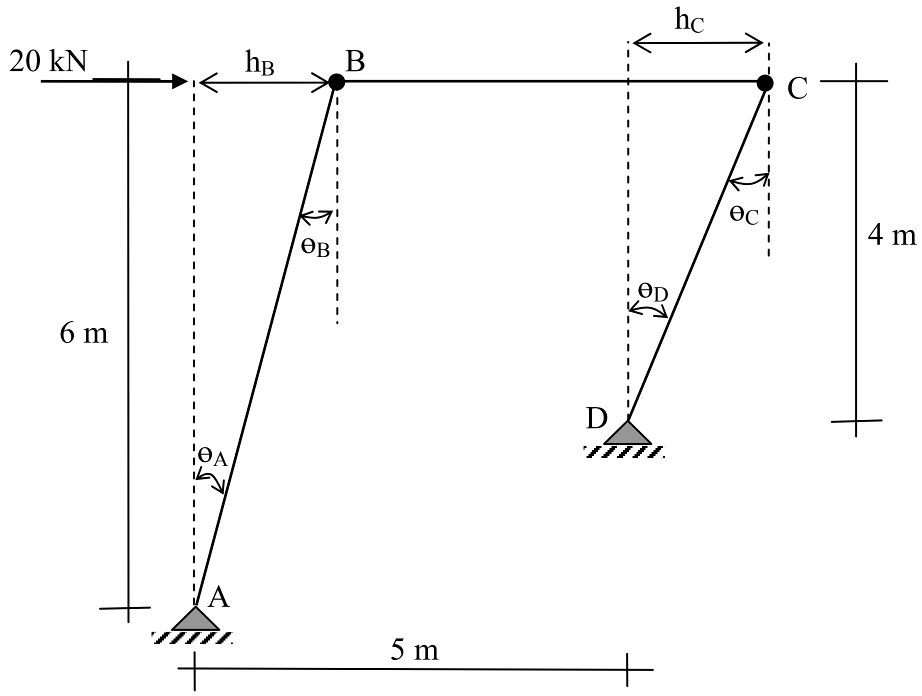
$$\theta_B = \left(\frac{h_E}{2.5}\right) = \theta \; ; \; h_E = 2.5\theta$$

$$\theta_C = \left(\frac{h_E}{2.5}\right) = \left(\frac{2.5\theta}{2.5}\right) = \theta$$

$$\theta_E = \theta_B + \theta_C = \theta + \theta = 2\theta$$

External work	**=**	**Internal work**
$(1/2 \times L \times h_E)(10)$	=	$Mp(\theta_B) + Mp(\theta_C) + Mp(\theta_E)$
$(1/2 \times 5 \times 2.5\theta)(10)$	=	$Mp(\theta) + Mp(\theta) + Mp(2\theta)$
$62.5\ (\theta)$	=	$4Mp(\theta)$
Mp	**=**	**15.63 kNm**

Mode failure 2 – Sway mechanism ABCD; plastic hinge occurred at B and C.



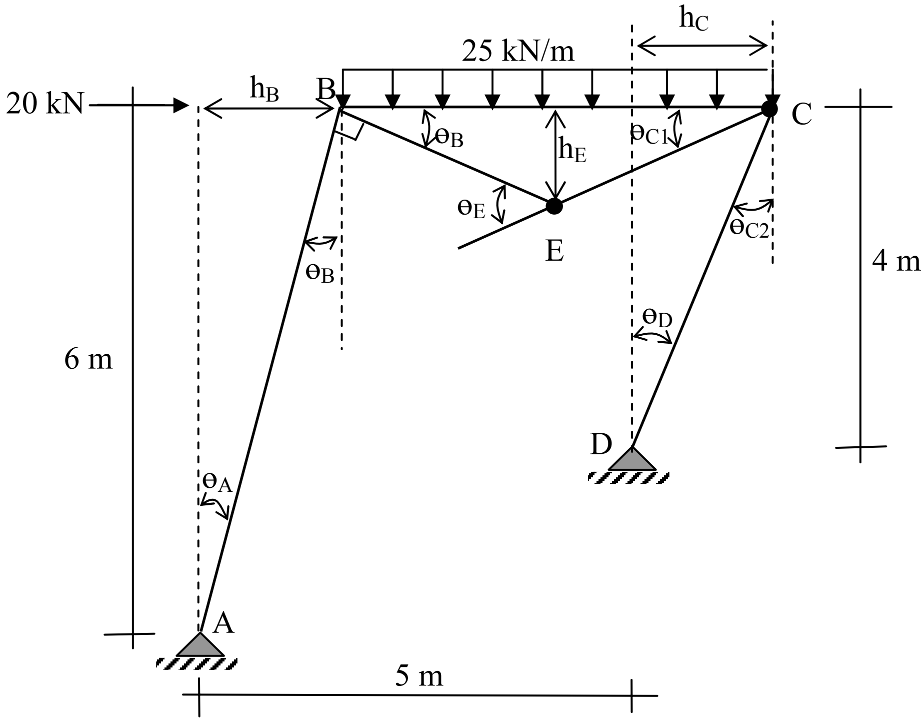
Assumes tan $\theta_A = \theta$; $\theta_A = \theta_B = \theta$; $\theta_C = \theta_D$; $h_B = h_C$

$$\theta_A = \left(\frac{h_B}{6}\right) = \theta \; ; \; h_B = 6\theta = h_C$$

$$\theta_D = \left(\frac{h_C}{4}\right) = \left(\frac{6\theta}{4}\right) = 1.5\theta$$

External work	=	Internal work
$(h_B)(20)$	=	$Mp(\theta_B) + Mp(\theta_C)$
$(6\theta)(20)$	=	$Mp(\theta) + Mp(1.5\theta)$
$120\,(\theta)$	=	$2.5Mp(\theta)$
Mp	**=**	**48 kNm**

Mode failure 3 – Combine mechanism ABCD; plastic hinge occurred at C and E.



Assumes $\tan\theta_A = \theta$; $\theta_A = \theta_B = \theta$; $h_B = h_C$

$\theta_A = \left(\frac{h_B}{6}\right) = \theta$; $h_B = 6\theta = h_C$

$h_E = 2.5\,(\theta_B) = 2.5\,\theta$

$\theta_{C1} = \theta_B = \theta$

$\theta_{C2} = \left(\frac{h_C}{4}\right) = \left(\frac{6\theta}{4}\right) = 1.5\theta$

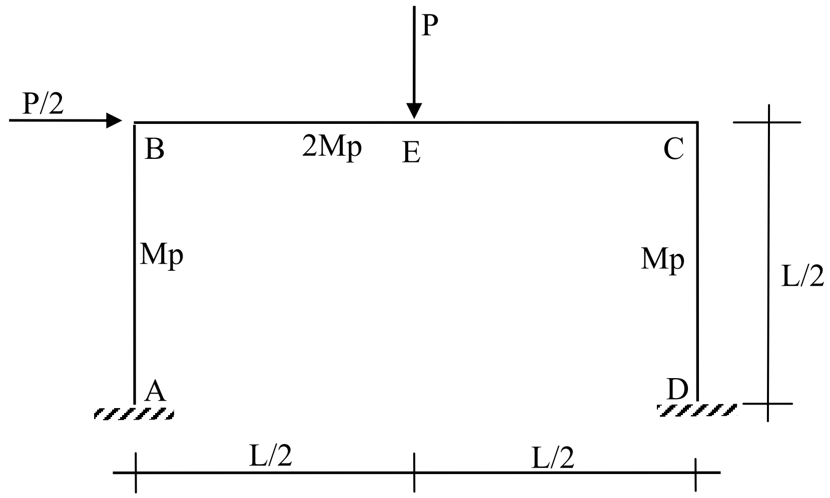
$\theta_E = \theta_B + \theta_{C1} = 2\theta$

External work	=	Internal work
$(h_B)(20) + (1/2 \times L \times h_E)(25)$	=	$Mp(\theta_{C1} + \theta_{C2}) + Mp(\theta_E)$
$(6\theta)(20) + (1/2 \times 5 \times 2.5\theta)(25)$	=	$Mp(2.5\theta) + Mp(2\theta)$
$120\,(\theta) + 156.25\,(\theta)$	=	$4.5\,Mp\,(\theta)$
$276.25\,(\theta)$	=	$4.5\,Mp\,(\theta)$
Mp	**=**	**61.4 kNm**

From three failure modes, the maximum plastic moment; **Mp max = 61.4 kNm**.

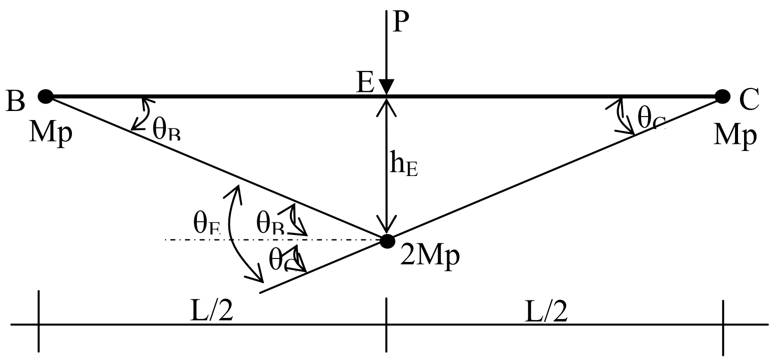
EXAMPLE 6.12

Determine collapse load, Pu from the frame shown and show all Mp for each modes.



Solution;

Mode failure 1 – Beam mechanism BC; plastic hinge occurred at B, C and E.



Assumes tan $\theta_B = \theta$

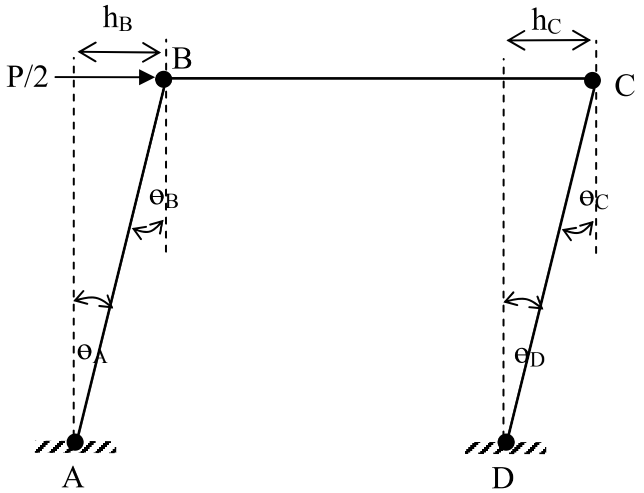
$$\theta_B = \left(\frac{h_E}{L/2}\right) = \theta\ ; \quad h_E = \frac{L}{2}\theta$$

$$\theta_C = \left(\frac{h_E}{L/2}\right) = \left(\frac{L/2}{L/2}\right)\theta = \theta$$

$$\theta_E = \theta_B + \theta_C = \theta + \theta = 2\theta$$

External work	**=**	**Internal work**
$P(h_E)$	=	$Mp(\theta_B) + Mp(\theta_C) + 2Mp(\theta_E)$
$P\left(\frac{L}{2}\theta\right)$	=	$Mp(\theta) + Mp(\theta) + 2Mp(2\theta)$
P	=	$\frac{16Mp}{L}$

Mode failure 2 – Sway mechanism ABCD; plastic hinge occurred at A, B, C and D.



Assumes $\tan\theta_A = \theta$; $\theta_A = \theta_B = \theta_C = \theta_D = \theta$; $h_B = h_C$

$$\theta_A = \left(\frac{h_B}{L/2}\right) = \theta\ ;\quad h_B = \frac{L}{2}\theta = h_C$$

External work = **Internal work**

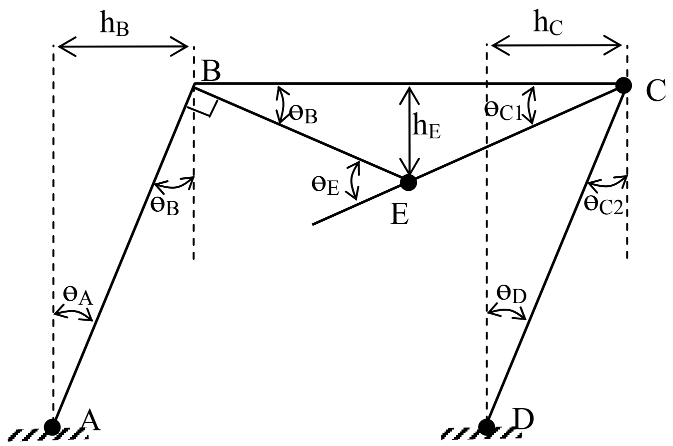
$$(h_B)(P/2) = Mp(\theta_A) + Mp(\theta_B) + Mp(\theta_C) + Mp(\theta_D)$$

$$\left(\frac{L}{2}\theta\right)\left(\frac{P}{2}\right) = Mp(\theta) + Mp(\theta) + Mp(\theta) + Mp(\theta)$$

$$\left(\frac{LP}{4}\right)(\theta) = 4Mp(\theta)$$

$$P = \frac{16Mp}{L}$$

Mode failure 3 – Combine mechanism ABCD; plastic hinge occurred at A, C, D and E.



Assumes tan $\theta_A = \theta$; $\theta_A = \theta_B = \theta$; $h_B = h_C$

$\theta_A = \left(\frac{h_B}{L/2}\right) = \theta$; $h_B = \frac{L}{2}\theta = h_C$

$h_E = \frac{L}{2}\theta$

$\theta_{C1} = \theta_B = \theta$

$\theta_{C2} = \theta_D = \left(\frac{h_C}{L/2}\right) = \left(\frac{L/2}{L/2}\right)\theta = \theta = \theta$

$\theta_E = \theta_B + \theta_{C1} = 2\theta$

External work = Internal work

$(P/2)(h_B) + P(h_E) = Mp(\theta_A) + Mp(\theta_{C1} + \theta_{C2}) + Mp(\theta_D) + Mp(\theta_E)$

$$\left(\frac{L}{2}\theta\right)\left(\frac{P}{2}\right) + P\left(\frac{L}{2}\theta\right) = Mp(\theta) + Mp(2\theta) + Mp(\theta) + 2Mp(2\theta)$$

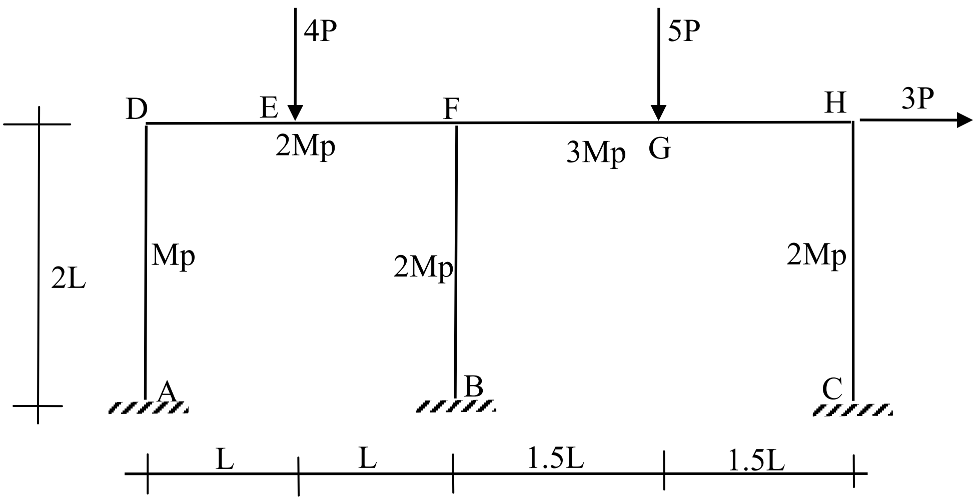
$$\left(\frac{3PL}{4}\right)(\theta) = 8Mp(\theta)$$

$$P = \frac{10.67Mp}{L}$$

Maximum collapse load, $Pu = \frac{10.67.Mp}{L}$ kN

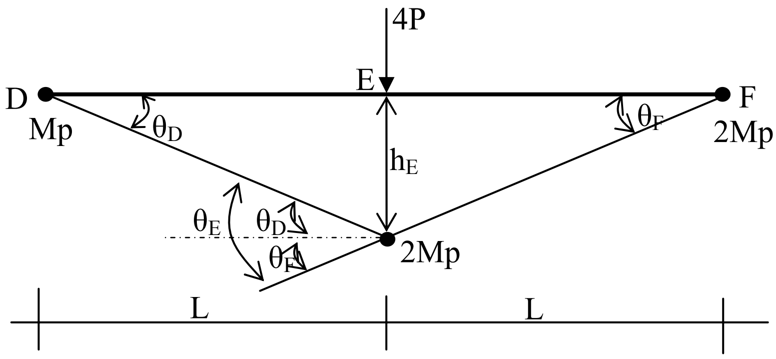
EXAMPLE 6.13

Determine collapse load, Pu from the frame shown and show all Mp for each modes.



Solution;

Mode failure 1 – Beam mechanism DF; plastic hinge occurred at D, E and F.



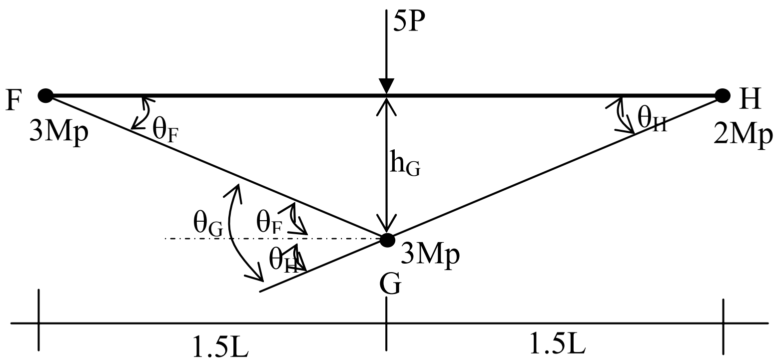
Assumes tan $\theta_D = \theta = \theta_F$

$\theta_D = \left(\frac{h_E}{L}\right) = \theta$; $h_E = L\theta$

$\theta_E = \theta_D + \theta_F = \theta + \theta = 2\theta$

External work	=	**Internal work**
$4P(h_E)$	=	$Mp(\theta_D) + 2Mp(\theta_E) + 2Mp(\theta_F)$
$4P(L\theta)$	=	$Mp(\theta) + 2Mp(2\theta) + 2Mp(\theta)$
$4LP(\theta)$	=	$7Mp(\theta)$
P	=	$\dfrac{1.75Mp}{L}$

Mode failure 2 – Beam mechanism FH; plastic hinge occurred at F, G and H.



Assumes tan $\theta_F = \theta = \theta_H$

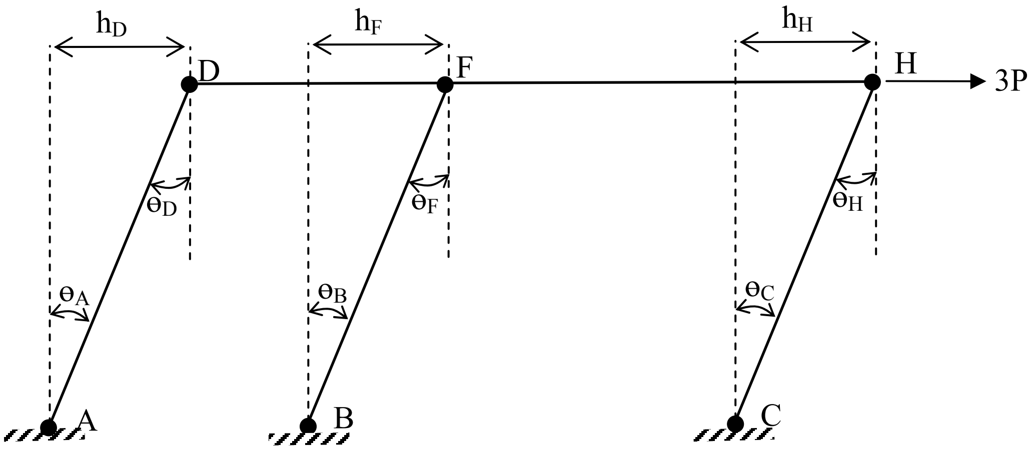
$\theta_F = \left(\frac{h_G}{1.5L}\right) = \theta$; $h_G = 1.5L\theta$

$\theta_G = \theta_F + \theta_H = \theta + \theta = 2\theta$

External work = Internal work

$$5P(h_G) = 3Mp(\theta_F) + 3Mp(\theta_G) + 2Mp(\theta_H)$$
$$5P(1.5L\theta) = 3Mp(\theta) + 3Mp(2\theta) + 2Mp(\theta)$$
$$7.5LP(\theta) = 11Mp(\theta)$$
$$P = \frac{1.47Mp}{L}$$

Mode failure 3 – Sway mechanism ADFHCB; plastic hinge occurred at A,B,C, D,F and H.



Assumes $\tan\theta_A = \theta = \theta_B = \theta_C = \theta_D = \theta_F = \theta_H$

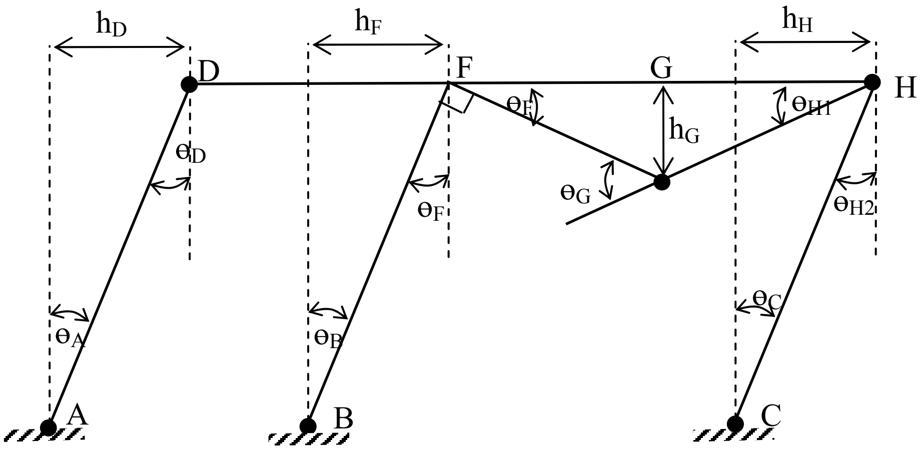
$$\theta_C = \left(\frac{h_H}{2L}\right) = \theta \; ; \quad h_H = 2L\theta$$

External work = Internal work

$$3P(h_H) = Mp(\theta_A + \theta_D) + 2Mp(\theta_B + \theta_F) + 2Mp(\theta_C + \theta_H)$$
$$3P(2L\theta) = Mp(2\theta) + 2Mp(2\theta) + 2Mp(2\theta)$$

$$6LP(\theta) = 10Mp(\theta)$$
$$P = \frac{1.67Mp}{L}$$

Mode failure 4 – Combine mechanism, Sway mechanism and beam mechanism FH; plastic hinge occurred at A,B,C, D,G and H.



Assumes tan $\theta_A = \theta = \theta_B = \theta_C = \theta_D = \theta_F = \theta_{H1} = \theta_{H2}$

$$\theta_F = \left(\frac{h_G}{1.5L}\right) = \theta\ ;\quad h_G = 1.5L\theta$$

$$\theta_C = \left(\frac{h_H}{2L}\right) = \theta\ ;\quad h_H = 2L\theta$$

$$\theta_G = \theta_F + \theta_{H1} = 2\theta$$

External work = Internal work

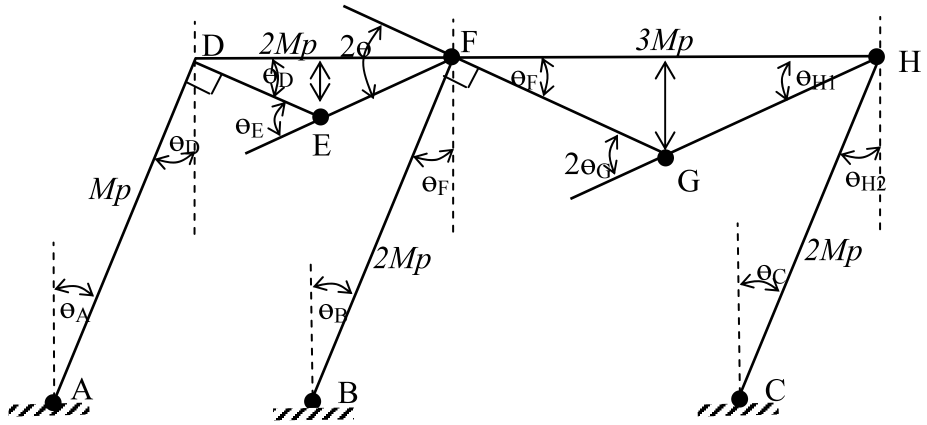
$$5P(h_G) + 3P(h_H) =$$
$$Mp(\theta_A) + Mp(\theta_D) + 2Mp(\theta_B) + 3Mp(\theta_G) + 2Mp(\theta_C) + 2Mp(\theta_{H1} + \theta_{H2})$$

$$5P(1.5L\theta) + 3P(2L\theta) = Mp(\theta) + Mp(\theta) + 2Mp(\theta) + 3Mp(2\theta) + 2Mp(\theta) + 2Mp(2\theta)$$

$$13.5LP(\theta) = 16Mp(\theta)$$

$$P = \frac{1.19Mp}{L}$$

Mode failure 5 – Combine mechanism, Sway mechanism and beam mechanism DF and FH;



Assumes tan $\theta_A = \theta = \theta_B = \theta_C = \theta_D = \theta_F = \theta_H$

$\theta_D = \left(\frac{h_E}{L}\right) = \theta$; $h_E = L\theta$

$\theta_F = \left(\frac{h_G}{1.5L}\right) = \theta$; $h_G = 1.5L\theta$

$\theta_E = \theta_D + \theta_F = 2\theta$

$\theta_G = \theta_F + \theta_{H1} = 2\theta$

External work	**=**	$4P(h_E) + 5P(h_G) + 3P(h_H)$
	=	$4PL\theta + 7.5PL\theta + 6PL\theta$
	=	$17.5\ Mp\theta$

Internal work

=

$$Mp(\theta_A) + 2Mp(\theta_E) + 2Mp(\theta_F) + 2Mp(\theta_B) + 3Mp(\theta_G) + 2Mp(\theta_C) + 2Mp(\theta_{H1} + \theta_{H2})$$

$= Mp(\theta) + 2Mp(2\theta) + 2Mp(2\theta) + 2Mp(\theta) + 3Mp(2\theta) + 2Mp(\theta) + 2Mp(2\theta)$

$= 23\ Mp\theta$

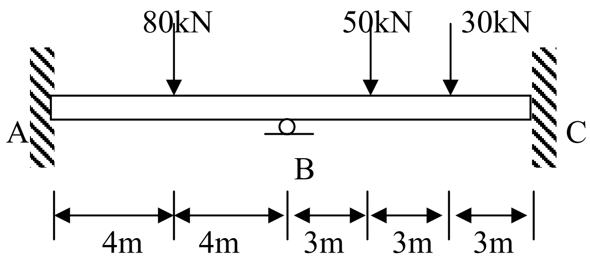
External work	**=**	**Internal work**
$17.5\ Mp\theta$	=	$23\ Mp\theta$
P	=	$\frac{1.31Mp}{L}$

From five failure modes, maximum collapse load occurred at mechanism no. 4;

$$\text{Maximum Pu} = \frac{1.19.Mp}{L}$$

TUTORIAL 6

1. The fixed support at the both end of beam and roller support located at the middle of the span. It carries point loads of 80kN, 50kN and 30kN. Determine the failure mode occurred in the beam and plastic moment, M_p.



2. (a) A continuous beam ABCDE is fixed at A and pinned at B and D is loaded as shown in Figure 7(a). Calculate the plastic moment *Mp* for each member of the beam.

[Ans: $M_{p(AB)}$ = 40kNm, $M_{p(BC)}$ = 109kNm]

(b) Calculate the plastic moment *Mp* for cross section as shown in Figure 7(b) if the yield stress for the material is σ_y = 225 N/mm^2.

[Ans: M_p = 62.5kNm]

(c) If the beam has a cross-section as shown in Figure(7b), state which part of the beam is safe.

(Final Sem2-2007/08)

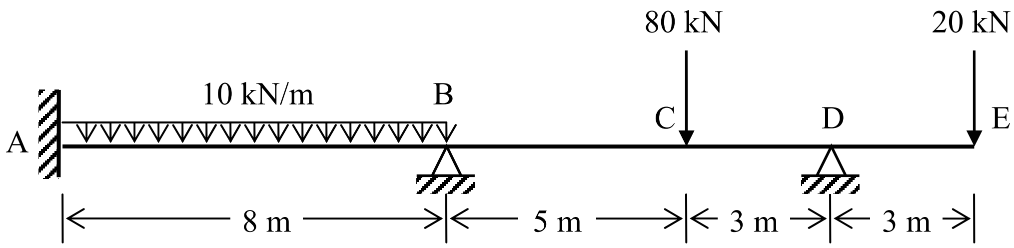


Figure 7a

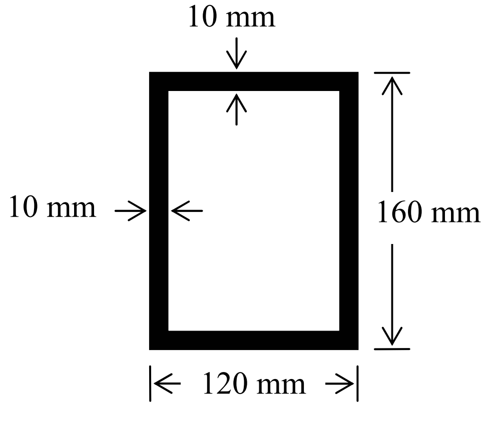


Figure 7b

3. (a) A steel beam with cross section as shown in Figure 8(a).

(i) Calculate plastic modulus, Z_p.

(ii) Determine steel yield stress if plastic moment of the cross section is 150 kNm

[**Ans:** $Z_p = 3037.5\ cm^3$, $\sigma_y = 49.38\ N/mm^2$]

(b) A continuous beam is loaded with various loads as illustrated in Figure 8(b). Determine the plastic moment for the beam.

[Ans: $M_{p(AB)} = 375kNm$, $M_{p(BC)} = 66.7kNm$, $M_{p(CB)} = 112.5kNm$]

(Final Sem1-2008/09)

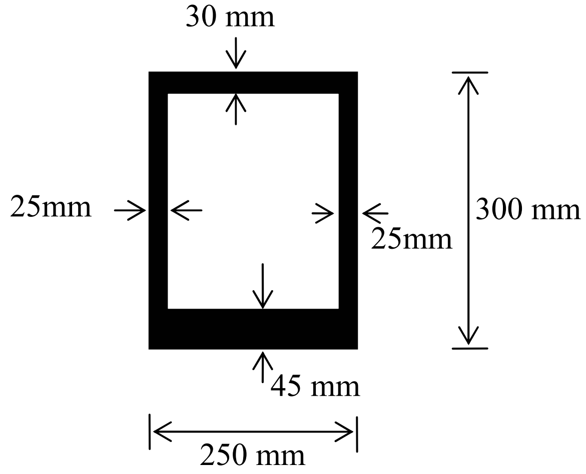


Figure 8(a)

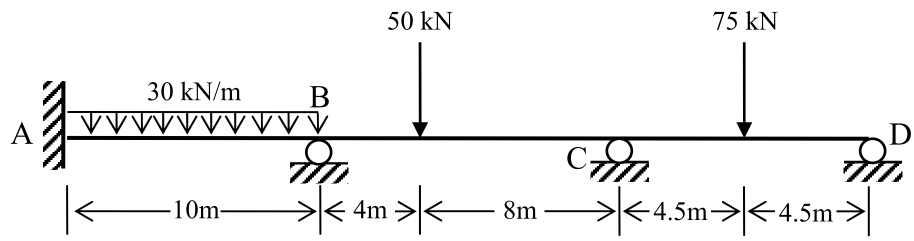


Figure 8(b)

4. (a) Figure 9(a) shows the characteristics stress-strain diagram for steel. Name five(5) points indicator from 1 to 5.

(b) Define the following terms;

(i) Hooke's Law

(ii) Modulus of Elasticity, E

(iii) Plastic hinge

(iv) Plastic moment

(v) Mechanism

(c) Figure 9(b) show the portal frame ABCD subjected to the point load acted at the joint B and two point loads acted at the span BC.

(i) Draw the Mode of Failure that might be happened.

(ii) Determine the maximum moment plastic occurred at Beam Mechanism BC.

[Ans: $M_{p(BC)}$ = 48.33kNm,63.33kNm]

(Final Sem2-2008/09)

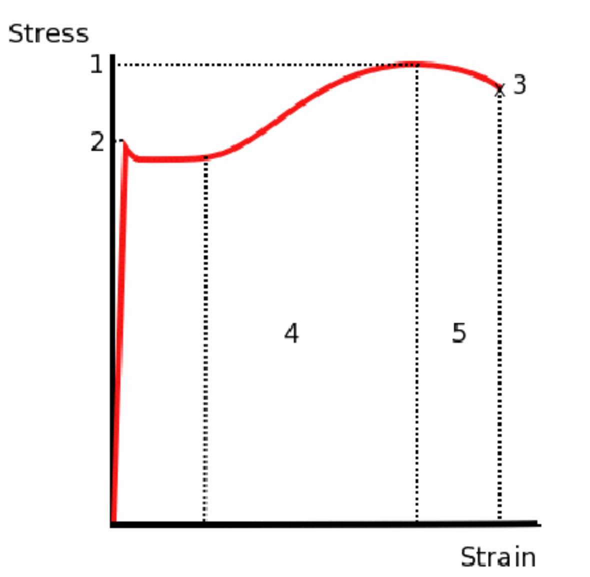


Figure 9(a)

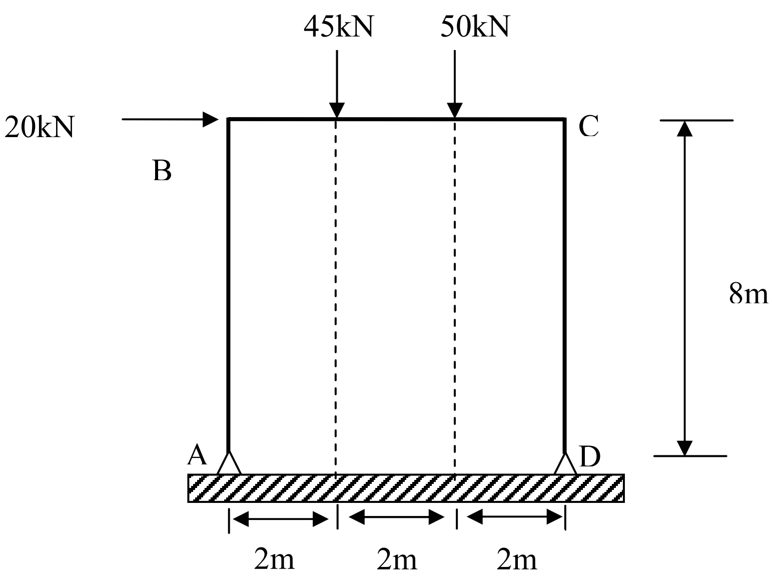


Figure 9(b)