Chapter 3
Stress in Beam

This topic explains the standard symbols for bending stresses. It also involves the position of neutral axis and second moment of area for standard cross section of beams and also determines stress in such beam cross-section.

On completion of this topic, students should be able to:

1. Define bending moment
2. Derive bending moment formulae
3. Calculate the stress in a beam due to bending
4. Find a location of neutral axis

3.1 Introduction

Beam, or flexural member, is frequently encountered in structures and machines. It is a member that subjected to loads applied transverse (sideways) to the long dimension, that causing the member to bend.

Transverse loading causes bending and it is very severe form of stressing a structure. The beat beam goes into tension (stretched) on one side and compression on the other. For example, a simply-supported beam loaded at its third-points will deform into the exaggerated bent shape shown in **Figure. 3.1**.

Figure 3.1 Example of a bent beam (loaded at its third points)

3.2 Classify Types of Beams

Before proceeding with a more detailed discussion of the stress analysis of beams, it is useful to classify some of the various types of beams and loadings encountered in practice as described in previous chapter.

Beams are frequently classified on the basis of supports or reactions such as pins, rollers, or smooth surfaces at the ends are called a simple beam. A simple support will develop a reaction normal to the beam, but will not produce a moment at the reaction.

If either or both ends of a beam projects beyond the supports, it is called a simple beam with overhang where as a beam with more than simple supports is a continuous beam. A cantilever beam is one in which one end is built into a wall or other support so that the built-in end cannot move transversely or rotate. The built-in end is said to be fixed if no rotation occurs and restrained if a limited amount of rotation occurs.

3.3 The Bending Theory Neutral

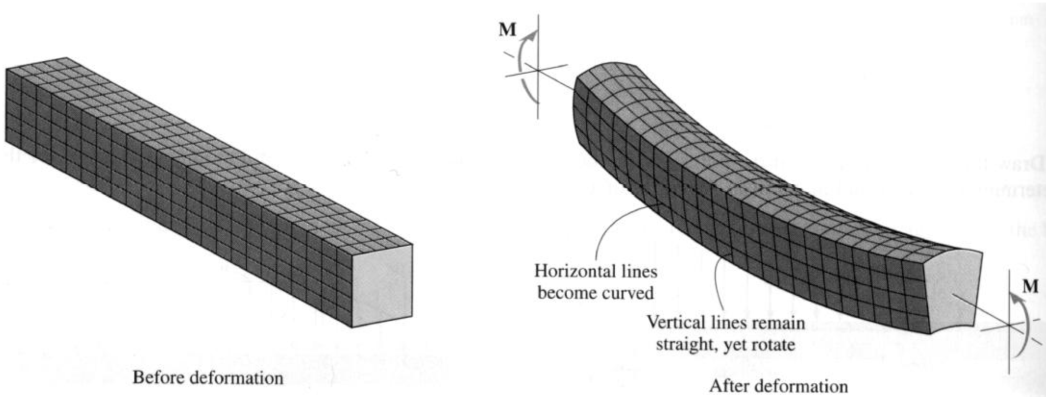
3.3.1 Neutral Axis (N.A)

This is the axis along the length of the beam which remains unstressed when it is bend. It is the region that separates the tensile forces from compressive force i.e. bending stress equal to zero. The position of the neutral axis must pass through the centroid of the section hence this position is important.

3.3.2 Derivation of flexural formula

To establish the bending stress formula several assumptions are used:-

1. The cross section of the beam is plane and must remain plane after bending **Figure 3.2**).
2. The beam's material is homogeneous and obey Hooke's Law
3. The material must be free from any resistance force and from impurities, holes, or grooves.
4. The bending moment of elasticity in tension must be the same for compression.
5. The beam has constant cross section.
6. The beam is subjected to pure bending.



Figures 3.2: Initially straight beam and the deformed bent beam

Due to the action of load beam will bend. Consider a beam that bent into an arc of a circle through angle ϴ radians. AB is on the neutral axis (will be the same length before and after bending). R is the radius of neutral axis **(Figures 3.3)**.

Beams in whatever shape will basically form a curve of x-y graph. One should be noted that the radius of curvature at any point on the graph is the radius of a circle.

The length of AB;

$$AB = R\Theta$$

Consider a layer of material with distance y from the N.A. The radius of this layer is R+y. The length of this layer which denoted by the line CD is;

$$CD = (R + y)\Theta$$

This layer is stretched because it becomes longer. Thus, it's been strained and strain, ε for this layer is;

$$\varepsilon = \frac{change\ in\ length}{original\ length} = \frac{CD - AB}{AB} = \frac{(R + y)\Theta - \mathrm{R}\Theta}{\mathrm{R}\Theta} = \frac{\mathrm{R}\Theta - \mathrm{y}\Theta - \mathrm{R}\Theta}{\mathrm{R}\Theta} = \frac{y}{R}$$

From Hooke's Law, modulus of elasticity, E

$$E = \frac{\sigma}{\varepsilon}$$

Substitute $\varepsilon = \frac{y}{R}$

$$E = \frac{\sigma R}{y}$$

Rearrange;

$$\frac{E}{R} = \frac{\sigma}{y}$$

** Stress and strain vary along the length of the beam depending on the radius of curvature.

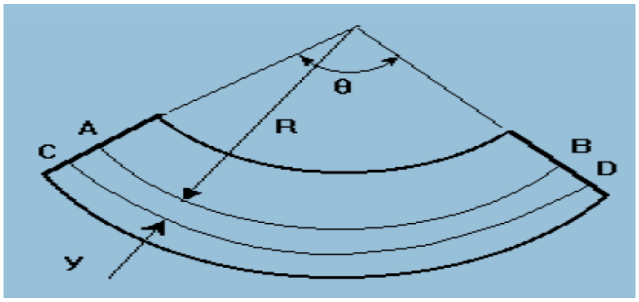


Figure 3.3

The fibre stress at a distance y from the neutral surface and is given by the expression

$$\sigma = \frac{My}{I}$$

At any section of the beam, the fibre stress will be maximum at the surface farthest from the neutral axis such that;

$$\sigma_{max} = \frac{Mc}{I} = \frac{M}{Z}$$

where c is the distance from NA to outer fiber (remember that the outer fibre will experienced maximum stress) and $Z = \frac{I}{c}$, is called the section modulus of the beam. Although the section modulus can be readily calculated for a given section, values of the modulus are often included in tables to simplify calculations.

3.3.3 Standard Sections

The value of I, second moment of area for a given section may be determined by formulae as in **Figure 3.4**. However, many beams are manufactured with standard section such as steel beams. From British Standard, the properties of standard steel beams and joists have been given in the standard code. The following formulae apply to standard shapes.

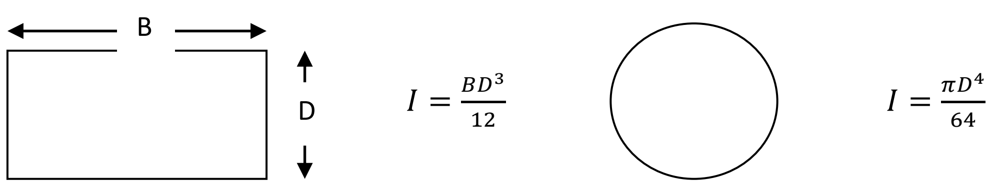
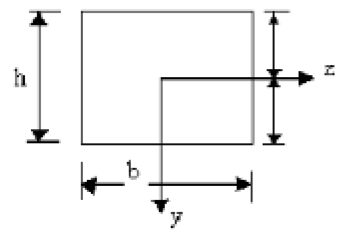


Figure 3.4

3.3.4 Elastic Section Modulus, Z

The section modulus (Z_e) is usually quoted for all standard sections and practically is of greater use than the second moment of area. Strength of the beam sections depends mainly on the second modulus. The section modulii of several shapes are calculated below (Figure 3.5);

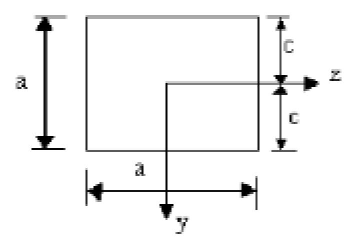
Rectangular Section



$I_z = bh^3/12$, $Ze = I_z/c$

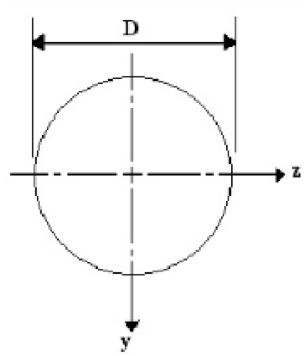
$$Z_e = \frac{I_z}{\frac{h}{2}} = (bh^3/2) / (2/h) = bh^2/6$$

Square Section



$I_z = a^4/12$
$Z_e = a^3/6$

Circular section



$I_z = \pi D^4/64$

$$Z_e = \frac{I_z}{(D/2)} = 2I_z /D = \pi D^3/32$$

Figure 3.5

3.4 Shear Stress in Beams

Although normal bending stresses appear to be of greatest concern for beams in bending, shear stresses do exist in beams when loads (i.e., transverse loads) other than pure bending moments are applied.

These shear stresses are of particular concern when the longitudinal shear strength of materials is low compared to the longitudinal tensile or compressive strength (an example of this is in wooden beams with the grain running along the length of the beam).

The effect of shear stresses can be visualized by considering a beam being made up of flat boards stacked on top of one another without being fastened together and then loaded in a direction normal to the surface of the boards. The resulting deformation will appear somewhat like a deck of cards when it is bent (see **Figure 3.6a**). The lack of such relative sliding and deformation in an actual solid beam suggests the presence of resisting shear stresses on longitudinal planes as if the boards in the example were bonded together as in **Figure 3.6b**, the resulting deformation will distort the beam such that some of the assumptions made to develop the bending strain and stress relations (for example, plane sections remaining plane) are not valid as shown in **Figure 3.7**.

Figure 3.6 Action of shear stresses in unbonded and bonded boards

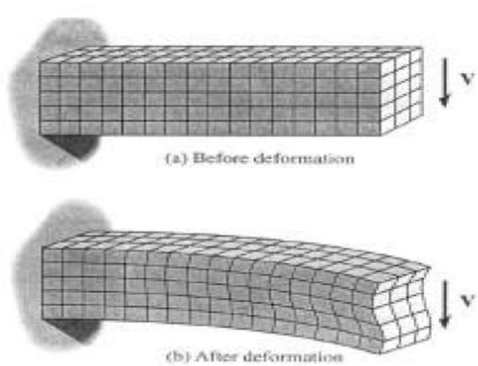


Figure 3.7 Distortion in a bend beam due to shear

The formula for the horizontal / longitudinal shear stress is:

$$\tau = \frac{VQ}{It} = \frac{VAy}{It}$$

Note that the formula is associated with a particular point in a beam and it is averaged across the thickness, t, and hence it is accurate only if t is not too great. For uniform cross sections, such as a rectangle, the shear stress takes on a parabolic distribution, with $\tau = 0$ at the outer fibre and $\tau = \tau_{max}$ at the neutral surface (where y = 0 and $\sigma = 0$) as shown in **Figure 3.8**. The maximum shear stress for certain uniform cross section geometries can be calculated as tabulated in **Figure 3.9**.

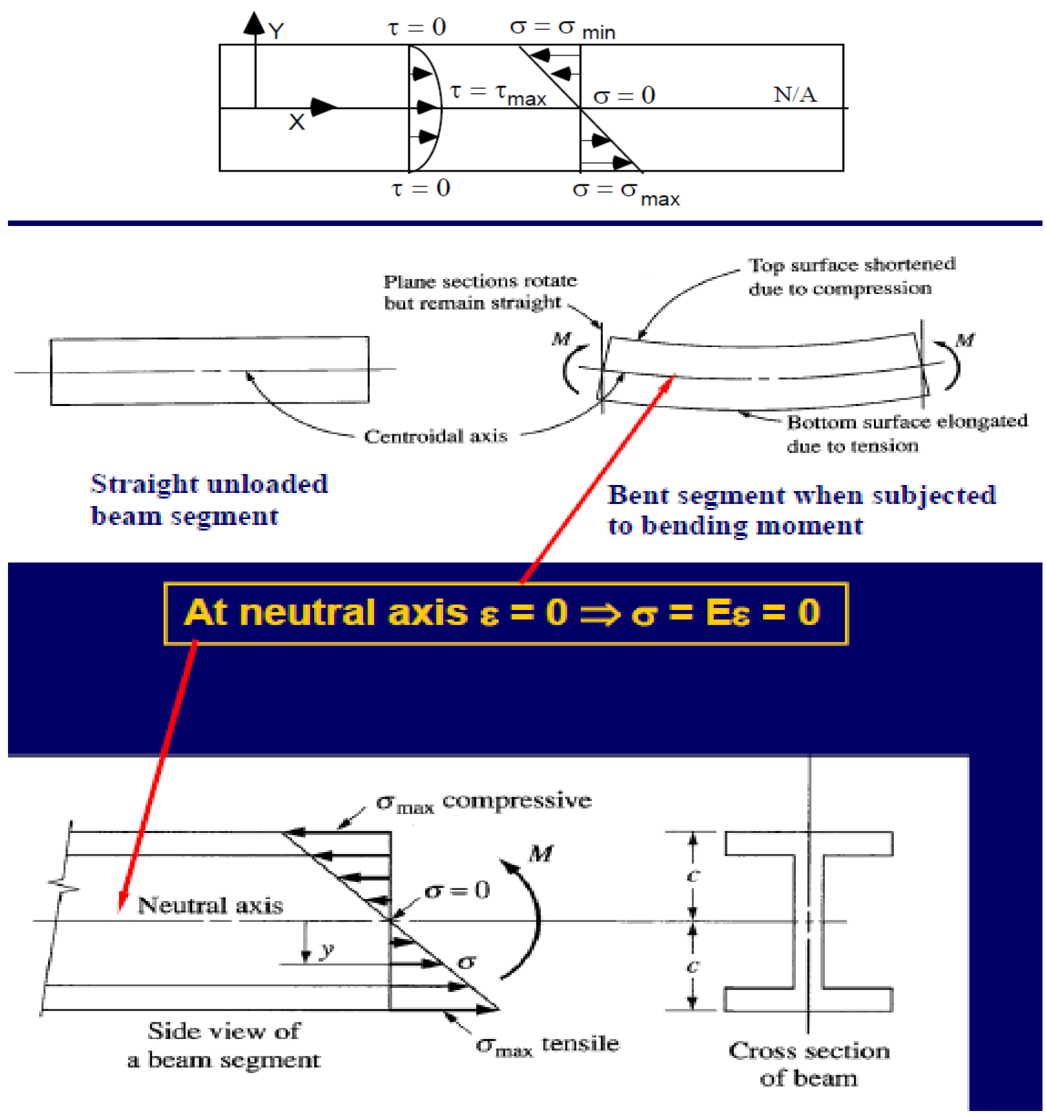


Figure 3.8: Shear and normal stress distributions in a uniform cross section beam

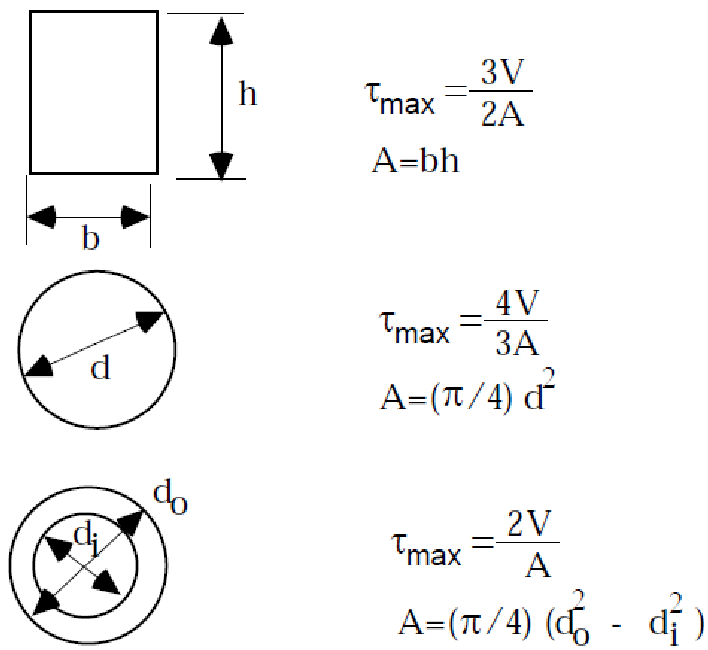


Figure 3.9

3.6 Pure Bending

Pure bending is a condition of stress where a bending moment is applied to a beam without the simultaneous application of axial, shear, or torsional forces. Beam that is subjected to pure bending means the shear force in the particular beam is zero, and no torsional or axial loads are presented.

Pure bending is also the flexure (bending) of a beam that under a constant bending moment (M) therefore pure bending only occurs when the shear force (V) is equal to zero since dM/dx= V. **Figure 3.10** shows an example of pure bending.

In pure bending the stresses vary linearly over the cross section of the beam, being zero at the neutral axis and maximum at the extreme fibers of the material to either side of the neutral axis. Such stresses are computed by formula as in equation below. However if the cross-sectional area of the beam changes suddenly (as in **Figure 3.11**), the stress distribution over the cross-section of the beam is

$$\sigma = k\frac{My}{I}$$

where K is the stress- concentration factor.

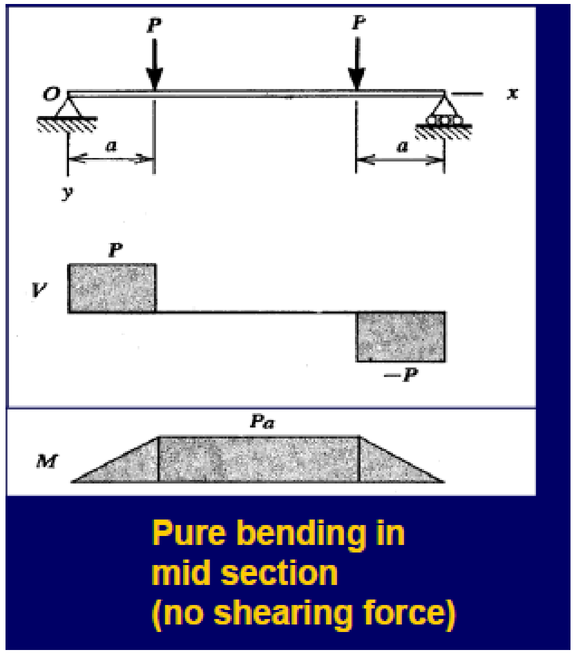


Figure 3.10

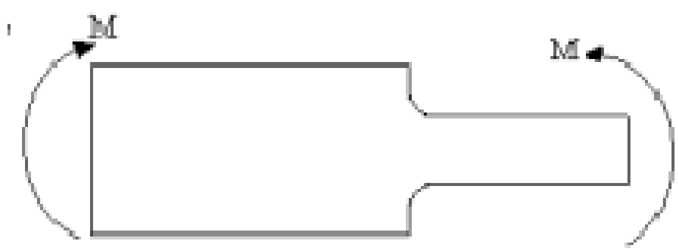


Figure 3.11

3.6 Efficient Use of Section in Bending

In the design of beam (the structure is a bending member!!! - as shown in **Figure 3.11**) the selected section must be strong in resisting the maximum bending moment as well as economical in weight per unit length. The condition of strength for beams in pure bending is satisfied provided that:

$$\sigma_{allowable} \geq \frac{M_{max}}{Z_e}$$

This equation also indicates that for a given allowable stress and a max bending moment the section modulus Z_e must not be less than the ratio ($M_{max} / \sigma_{allowable}$). If the allowable stress of the material in tension is the same as in compression the use of a section which is symmetrical about the neutral axis is preferred and the material from which the beam is made should be ductile. Structural steel is a good example of a ductile material.

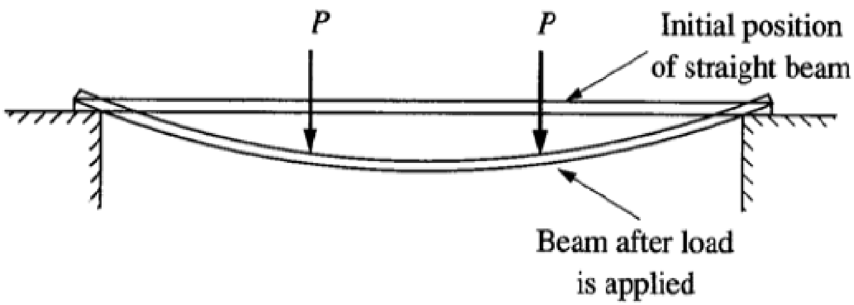


Figure 3.12

The next condition to be satisfied is the economy in weight of the beam. This is accomplished by distributing as much of the area of the cross- section as far as possible from the neutral axis of the section.

Discuss about this by giving examples for different shape of cross-sections.

3.7 Examples

Example 3.1

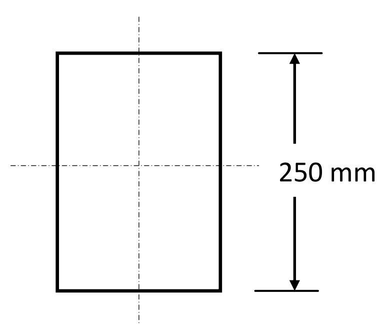
A 250 mm (depth) x 150 mm (width) rectangular beam is subjected to maximum bending moment of 750 kNm , find;

i. The maximum stress in the beam
ii. The value of the longitudinal stress at a distance of 65 mm from the top surface of the beam.

Solution:

i. maximum stress in the beam;

$$\sigma = \frac{My}{I}$$



Distance of N.A. from the top surface of the beam;

$$y = \frac{h}{2} = \frac{250}{2} = 125\ mm\ = 0.125\ m$$

Moment of inertia, ***I***

$$I = \frac{bh^3}{12} = \frac{150 \times 250^3}{12} = 195312500\ mm^4\ = 0.0001953\ m^4$$

Using relationship

$$\sigma = \frac{My}{I} = \frac{750 \times 10^3 \times 0.125}{0.0001953} = 4.8 \times 10^8 Nm^{-2}$$

ii. longitudinal stress at a distance of 65 mm from the top surface

Using relationship

$$\sigma = \frac{My}{I} = \frac{My_1}{I}$$

Distance y = y1 = 60 mm = 0.06 m

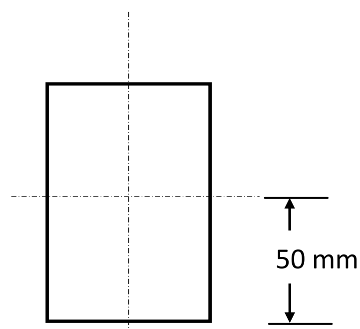
$$\sigma = \frac{My_1}{I} = \frac{750 \times 10^3 \times 0.06}{0.0001953} = 230\ \boldsymbol{MNm^{-2}}$$

Example 3.2

A beam has a rectangular cross section 80 mm wide x 100 mm deep. It is subjected to maximum bending moment of 15 kNm. The beam is made from metal that has a modulus of elasticity of 180 GPa. Calculate the maximum stress on the section.

Solution:

iii. maximum stress in the beam



$$\sigma = \frac{My}{I}$$

$$I = \frac{bh^3}{12} = \frac{80 \times 100^3}{12} = 6.667 \times 10^6\ mm^4$$

Distance of N.A. from the outer surface (maximum stress!!!) of the beam, y = 50 mm.

$$\sigma = \frac{My}{I} = \frac{15 \times 10^3 \times 0.05}{6.667 \times 10^6} = 112.5 \times 10^6\ \boldsymbol{Nm^{-2}}$$

Example 3.3

A symmetrical section with 200 mm, deep has a moment of inertia of 2.26x10^5 m^4 about its N.A. Find the longest span over which , when simply supported the beam would carry a uniformly distributed load 4 kN/m run without the stress due to bending exceeding 125 MN/m

Solution:

Using relationship

$$\sigma = \frac{My}{I}$$

Maximum moment, M when $\sigma = \sigma_{max} = 125\frac{MN}{m} = 125 \times 10^6 \, N/m$

$$M = \frac{\sigma I}{y} = \frac{125 \times 10^6 \times 2.26\text{x}10^5}{0.1} = 28.25 \times 10^3 Nm$$

For simply supported beam, the maximum bending moment due to uniformly distributed loads is;

$$M = \frac{wL^2}{8} = M_{max} = 28.25 \times 10^3 Nm$$

$$28.25 \times 10^3 Nm = \frac{wL^2}{8} = \frac{4 \times 10^3 L^2}{8}$$

$$28.25 = 0.5L^2$$

$$L = 7.5 \, m$$

Example 3.4

Find the dimensions of a timber beam with length 8 m to carry a brick wall of 200 mm thick and 5 m high. If the density of brick work is1850 Kg/m^3 and the maximum permissible stress is limited to 7.5 MN/m^2 given that the depth of beam is twice the width.

Solution:

Total weight of the wall = W

$$W = span \times thickness\ of\ the\ wall \times height\ of\ the\ wall \times density\ of\ brick$$

$$W = 8 \times 0.2 \times 5 \times 1850 \times 9.81 = 0.145 \, kN$$

For simply supported beam with 8 m length, the maximum bending moment,

M_{max} = 0.29MNm

Using relationship

$$M = \frac{\sigma I}{y}$$

Given the depth of the beam is twice the width, let says depth is b thus width is $2b$. For rectangular beam shape, its centroid located at the mid depth, thus the distance of the outer edge from the centriod is b

$$M_{max} = 0.29 = \frac{7.5 \times \frac{b(2b)^3}{12}}{b}$$

$$depth, b = 0.39\,m \quad and\ the\ width\ 2b = 0.77m$$

Example 3.5

Solution

Maximum Internal Moment. The maximum internal moment in the beam, $M = 22.5\text{ kN}\cdot\text{m}$, occurs at the center as shown on the bending moment diagram, Fig. 6–28*c*. See Example 6.3.

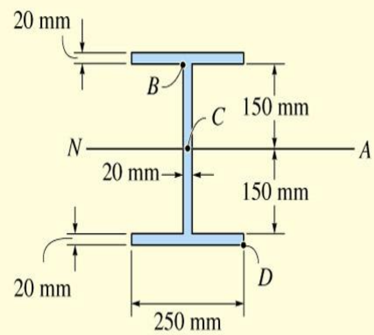
Section Property. By reasons of symmetry, the centroid *C* and thus the neutral axis pass through the midheight of the beam, Fig. 6–28*b*. The area is subdivided into the three parts shown, and the moment of inertia of each part is computed about the neutral axis using the parallel-axis theorem. (See Eq. A–5 of Appendix A.) Choosing to work in meters, we have

$$I = \Sigma(\bar{I} + Ad^2)$$

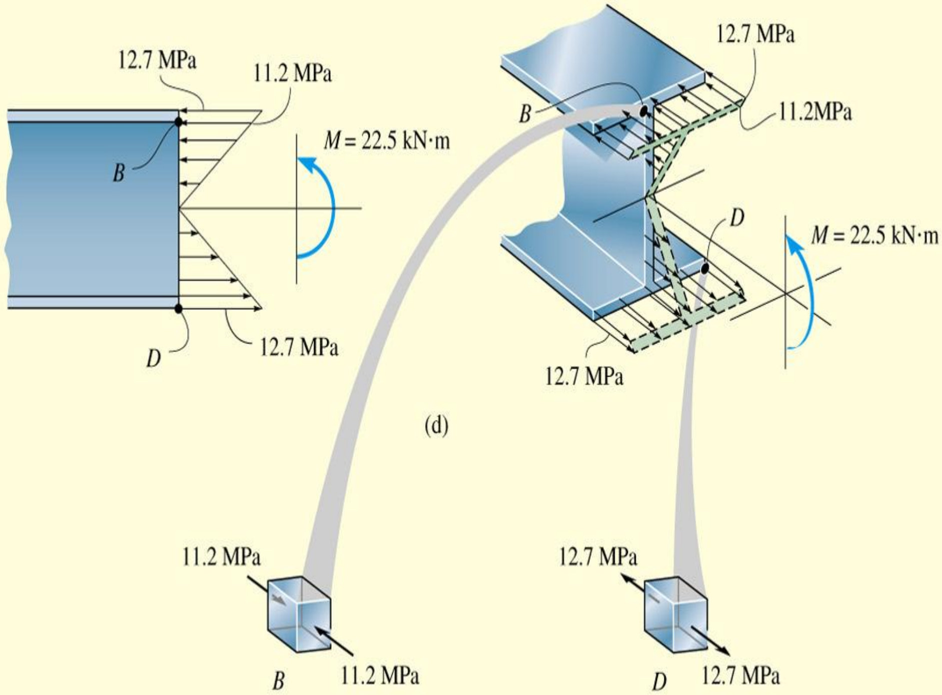
$$= 2\left[\frac{1}{12}(0.25\text{ m})(0.020\text{ m})^3 + (0.25\text{ m})(0.020\text{ m})(0.160\text{ m})^2\right]$$

$$+ \left[\frac{1}{12}(0.020\text{ m})(0.300\text{ m})^3\right]$$

$$= 301.3(10^{-6})\text{ m}^4$$



(b)



(d)

(e)

Bending Stress. Applying the flexure formula, with $c = 170$ mm, the absolute maximum bending stress is

$$\sigma_{\max} = \frac{Mc}{I}; \qquad \sigma_{\max} = \frac{22.5\ \text{kN}\cdot\text{m}(0.170\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = 12.7\ \text{MPa} \qquad \textit{Ans.}$$

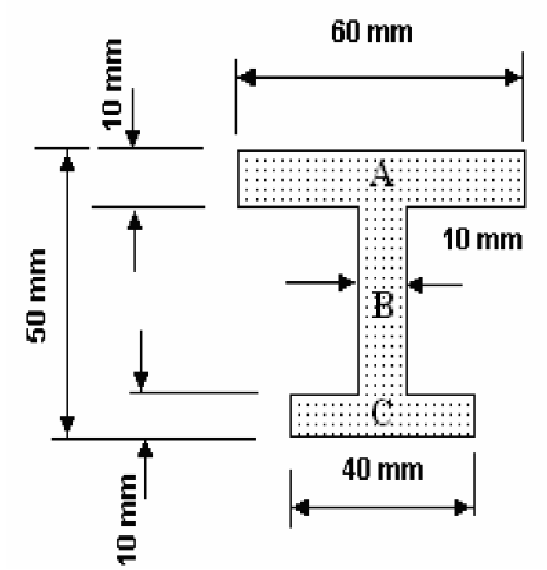
Two-and-three-dimensional views of the stress distribution are shown in Fig. 6–28*d*. Notice how the stress at each point on the cross section develops a force that contributes a moment $d\mathbf{M}$ about the neutral axis such that it has the same direction as $\mathbf{M}$. Specifically, at point B, $y_B = 150$ mm, and so

$$\sigma_B = \frac{My_B}{I}; \quad \sigma_B = \frac{22.5\ \text{kN}\cdot\text{m}(0.150\ \text{m})}{301.3(10^{-6})\ \text{m}^4} = 11.2\ \text{MPa}$$

The normal stress acting on elements of material located at points B and D is shown in Fig. 6–28*e*.

Example 3.6

Calculate the stress on the top and bottom of the section shown when the bending moment is 300 N m. Draw the stress distribution.



First calculate the second moment of area using the tabular method that you should already know. Divide the shape into three sections A, B and C. First determine the position of the centroid from the bottom edge.

	Area	$\bar{y}$	$A\bar{y}$
A	600 mm^2	45 mm	27 000 mm^3
B	300 mm^2	25 mm	7500 mm^3
C	400 mm^2	5 mm	2000 mm^3
Totals	1300 mm^2		365000 mm^3

For the whole section the centroid position is $\bar{y}$=365000/1300 = 28.07 mm

Now find the second moment of area about the base. Using the parallel axis theorem.

	$BD^3/12$	$A\bar{y}^2$	$I = BD^3/12 + A\bar{y}^2$
A	$60 \times 10^3/12 = 5000$ mm^4	$600 \times 45^2 = 1215000$	1220000 mm^4
B	$10 \times 30^3/12 = 22500$ mm^4	$300 \times 25^2 = 187500$	210000 mm^4
C	$40 \times 10^3/12 = 3333$ mm^4	$400 \times 5^2 = 10000$	13333 mm^4
			Total = 1443333 mm^4

The total second moment of area about the bottom is 1443333 mm4
Now move this to the centroid using the parallel axis theorem.

$$I = 1443333 - A\bar{y}^2 = 1443333 - 1300 \times 28.08^2 = 418300 \text{ mm}^4$$

Now calculate the stress using the well known formula $\sigma_B = My/I$

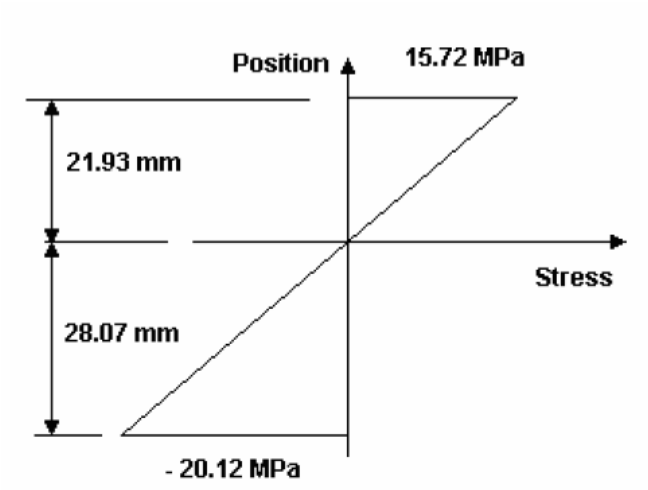
Top edge y = distance from the centroid to the edge = 50 – 28.08 = 21.93 mm

$\sigma_B = 300 \times 0.02192/418.300 \times 10^{-9} = 15.72 \times 10^6$ Pa or 15.72 MPa (Tensile)

Bottom edge y = $\bar{y}$ =28.07 mm

$\sigma_B = 300 \times 0.02808/418.300 \times 10^{-9} = 20.14$ MPa (Tensile)

The stress distribution looks like this.



Assignment

A symmetrical I-section beam is 60 mm deep with a second moment of area of 663 x 10^{-9} m^4 and a cross sectional area of 1600 mm^2. It is subject to a bending moment of 1.2 kNm and an axial force of 25 kN (tension). Find the position of the neutral axis of the beam.

(Stresses 69.92/-38.67 and NA 38.6 mm from the tensile edge)

Open completion of this topic, students should be able to;

1. Define a beam
2. Recognized different types of beams
3. Define and Derive bending moment formulae
4. Calculate the stress in beam due to bending.
5. Solve problems involving both bending and direct stress

6. Find the position of N.A
   Students should understand the basic of principles of moment, shear force, stress and moment of area.

Extra explanation:

http://www.youtube.com/watch?v=9C9GFs5AK4c